Numerical Modeling of the Acoustics of Low Density Fibrous Media Having a Distribution of Fiber Sizes

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Introduction

Focus

• Low density, polymeric, fibrous medium
  – Blown microfibers with broad fiber size distribution
  – Staple fiber component with larger but narrow size distribution

Airflow Resistivity

• Considered the most important macroscopic quantity for acoustics
• Literature typically only considers a single uniform fiber size
• Tarnow, for example, considers uniform size with random 2-D spacing
• Authors extended theory to distributions of fiber size (InterNoise 2017)

Acoustic Properties

• Derived from resistivity using limp fluid models such as JCA
  – Method doesn’t explicitly account for range of fiber sizes
• Present work to determine
  – Reliability of resistivity calculation
  – Applicability for media with broad fiber size distribution (work in progress)
Modeling Approach

Numerical Tools

- GeoDict with FlowDict by Math2Market (a Fraunhofer Institute spinoff)
  - Fiber geometry generation, including fiber size distributions and 3D orientation
  - Flow resistance calculation (CFD)
  - AcoustoDict with DiffuDict can also calculate other JCA parameters

- 3M Proprietary Software
  - Import GeoDict geometry and create a COMSOL model

- COMSOL Multiphysics
  - Flow resistance calculation (CFD)
  - Diffusion calculation (for remaining JCA parameters)
  - Visco-thermal acoustic calculation (rigid fibers or fluid-structure interaction for limp fibers)
**Tarnow Perpendicular-Random Model**

**Tarnow 1996**

**Inputs**
- Fiber size (radius): $r$
- Fiber bulk density: $\rho_{lb}$
- Solid material density: $\rho$

**Output**
- Fiber mean spacing: $b_{1/2} = \frac{\pi}{\sqrt{r^{1/2} / C}}$
- Solidity: $C = \frac{\rho_{lb}}{\rho}$
- AFR: $\sigma = \frac{4\pi \rho_{lb} / b_{1/2}}{[0.640 \ln(1/C) + C - 0.737]}$ (Based on Tarnow model)

**Steps**
- **Step 1**: $C$ calculation based on $\rho_{lb}$ and $\rho$
- **Step 2**: $b_{1/2}$ calculation based on $r$ and $C$
- **Step 3**: $\sigma$ calculation based on $C$ and $b_{1/2}$

Tarnow Voronoi Cells: for modeling the mean spacing $b_{1/2}$ distributed around each fiber cylinder. Fiber F is relocated to centroid G.
Tarnow Verification – Uniform fiber size

2D Results
- Regular lattice
  - Exact match to equations
- Random fibers in 2D
  - Lower resistivity is for parallel case
  - Tarnow assumed fibers centered within Voronoi cells

3D Extensions
- Resistivity vs. air flow angle
  - Nonlinear variation (more data needed)
- Isotropic 3D distribution
  - Between parallel and perpendicular
- Transverse isotropic 3D distribution
  - Much like perpendicular
**Xue – InterNoise 2017**

**Inputs**
- Fiber mean radii: $r_{↓1}$, $r_{↓2}$, distribution parameters
- Fiber bulk density: $\rho_{↓b}$
- Component weight fractions: $X_{↓1}$, $X_{↓2}$
- Solid material densities: $\rho_{↓1}$, $\rho_{↓2}$

**Output**
- AFR: $\sigma = 4\pi \eta / b_{↑2} \left[ 0.640 \ln \left( \frac{1}{C} \right) + C - 0.737 \right]$ (Based on Tarnow model)

The modifications were made only on Step 1 and Step 2.

- **Step 1**: $C$ calculation based on $\rho_{↓b}$, $X_{↓1}$, $X_{↓2}$
- **Step 2**: $b_{↑2}$ calculation based on $r_{↓1}$, $r_{↓2}$, distribution parameters and $C$
- **Step 3**: $\sigma$ calculation base on $C$ and $b_{↑2}$

**Equations**
- Fiber bulk density: $\rho_{↓b}$
- Component weight fractions: $X_{↓1}$, $X_{↓2}$
- Solid material densities: $\rho_{↓1}$, $\rho_{↓2}$
- Solidity: $C = \frac{X_{↓1} \rho_{↓1}}{\rho_{↓b}} + \frac{X_{↓2} \rho_{↓2}}{\rho_{↓1}}$
- Fiber mean spacing: $b_{↑2} = \pi / C (\sum_{p=1}^{↑j} n_{↓1,p} r_{↓1,p}^{↑2} + \sum_{q=1}^{↑k} n_{↓2,q} r_{↓2,q}^{↑2}) / (\sum_{p=1}^{↑j} n_{↓1,p} + \sum_{q=1}^{↑k} n_{↓2,q})$
- AFR: $\sigma = 4\pi \eta / b_{↑2} \left[ 0.640 \ln \left( \frac{1}{C} \right) + C - 0.737 \right]$ (Based on Tarnow model)
Flow Resistance for 1 mm³ volume

• Broader distributions have lower flow resistance
  – Larger fibers appear to reduce FR more than small fibers increase it
• Tarnow calculation vs. GeoDict
  – Shows similar trends
  – Tarnow has about 18-20% lower resistance
  – GeoDict is overpredicting dP compared to COMSOL
    (3 cases compared – could be due in part to 1 µm voxels and boundary effects)
Broad Fiber Size Distribution

Measurement Difficulties

- Micro-CT scanning
  - Biased to larger fibers and clumps
  - Algorithm broadens distribution artificially
- SEM Images
  - Automatic fiber counting
    - Need a clean section of fiber
    - May count same fiber several times
    - May count bundles as a single fiber
JCA Parameters from COMSOL

Geometry
- Porosity
- Thermal characteristic length

Laplace Equation in Air Space only
- Inviscid flow, thermal conduction, or electric conduction
  - Apply a potential difference and compute current
- Tortuosity factor and tortuosity
- Viscous characteristic length

\[ \Lambda_{\text{therm}} = 2 \int \frac{Vol \uparrow \downarrow dV}{\int \text{ Surf} \uparrow \downarrow dS} = \frac{2V}{S} \quad \phi = \frac{V}{\Delta X \Delta Y \Delta Z} \]

\[ J = \int \text{ Inlet} \uparrow \downarrow J_{\downarrow z} dS = \frac{1}{V} \int \text{ Vol} \uparrow \downarrow J_{\downarrow z} dV \]

\[ \alpha_{\downarrow \infty} = \phi \Delta V \Delta X \Delta Y C/J/\Delta Z \approx V \int \text{ Vol} \uparrow \downarrow J J dV \]

\[ \Lambda_{\text{visc}} = 2 \int \frac{Vol \uparrow \downarrow |J| \uparrow 2 dV}{\int \text{ Surf} \uparrow \downarrow |J| \uparrow 2 dS} \]

\[ \tau = \sqrt{\alpha_{\downarrow \infty}} \]
JCA Parameters from COMSOL

Viscous Flow

- Flow resistance and resistivity
- Low frequency tortuosity factor
  - $\alpha_0 > \alpha_\infty$ due to $v=0$ on fiber surfaces

\[
\nu_{ave} = \frac{1}{\Delta X \Delta Y} \int \text{Inlet} \uparrow u \, dS \\
\sigma = \frac{dP}{\phi \Delta Z} \nu_{ave} =
\]

\[
\alpha_0 = V \int \text{Vol} \uparrow \nu \nu \frac{dV}{(\int \text{Vol} \uparrow \nu \downarrow z \frac{dV}{})^{\frac{1}{2}}}
\]

Pressure Field
Velocity Mag.
Direct Modeling of the Thermo-Acoustic fields

- Computes acoustic pressure and velocity fields along with fiber motion.
- 4-mic impedance tube equations \( (1\text{-load}) \) using virtual pressure points within model.
- Allows computation of wavenumber and complex speed of sound, density, and bulk modulus.
- Requires HPC-system: used one node with 28 cores and 1TB RAM.

Pressure Field  Velocity Field  Structural Motion colored by Pressure Gradient
GeoDict Fiber Size Distribution

Fiber size distribution generated by GeoDict

<table>
<thead>
<tr>
<th>Solidity</th>
<th>Uniform Fiber Flow Resistance</th>
<th>Broad Distribution Flow Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>13,303 Rayl</td>
<td>10,746 Rayl</td>
</tr>
<tr>
<td>2%</td>
<td>33,062 Rayl</td>
<td>26,707 Rayl</td>
</tr>
<tr>
<td>3%</td>
<td>57,643 Rayl</td>
<td>46,562 Rayl</td>
</tr>
</tbody>
</table>

Tarnow calculation for model fiber distributions
Compute complex material properties

- 200×200×1000 μm Volume
- 3 Realizations at each solidity
- Higher solidity shifts curves to higher frequencies

10μm Fibers – Sound Phase Speed
Variability of 3 realizations
**COMSOL Acoustic Model with Structural Motion (FSI)**

**Compute complex material properties**
- Size distribution increases case-to-case variability
- Suggests the need for larger models or averaging many cases

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**Fiber Distribution – Sound Phase Speed Variability of 3 instances**

2% Solidity
JCA (Limp) vs. Direct Acoustic Properties – Uniform Fibers

Wave Number – *Good*
Density – *Excellent*
Speed of Sound – *Fair*
Bulk Modulus – *Poor*

Solid curves are JCA calculation
Markers are direct model results

2% Solidity
Wave Number – Good
Density – Good
Speed of Sound – Fair
Bulk Modulus – Poor
All are a little worse than uniform fiber size.

Solid curves are JCA calculation
Markers are direct model results

2% Solidity
JCA (Limp) vs. Direct Acoustic Properties – Broad Fiber Size Distribution

Wave Number – Good
Density – Good
Speed of Sound – Fair
Bulk Modulus – Better at high frequencies

3% Solidity appears to be a bit better behaved, also less variable.

Solid curves are JCA calculation
Markers are direct model results

3% Solidity
Attenuation per Wavelength

- Black line: Broad 10um, 2% Solidity
- Red line: Broad 10um, 3% Solidity
- Green line: Uniform 10um, 2% Solidity

Axes:
- Y-axis: Attenuation per Wavelength [-]
- X-axis: Frequency [Hz]

Frequency range: 100 to 10000 Hz

Values range: -4 to 4
Bonding changes fiber motion significantly
- Individual fibers move more or less, depending on their diameter and orientation
- Bonded fibers move in unison
- *Simple test case of 500×500×200 μm at 3% solidity*
Conclusions

Results

• Extended Tarnow calculations match CFD calculations reasonably well.
• Fiber size distributions are difficult to obtain.
• JCA parameters can be obtained from finite element models.
• Acoustic equivalent fluid properties can be calculated directly using FEA.
  – Much variability for small systems with low solidity.
  – Results don’t necessarily match JCA calculations.
  – Large computers are needed for modeling fine fibers.
• Intersecting or bonded fibers tend to move in unison.

Next steps

• 2-Load method which doesn’t require/assume a symmetric sample.
• Determine difference between COMSOL and FlowDict pressure drop calculations.
• Determine acoustic effect of individual vs. bonded fibers.
• Currently adiabatic or isothermal fibers, can be extended to included heat transfer in fibers.
• Consideration of fiber length distribution (after c. Perrot’s presentation).