FRACTIONAL DERIVATIVES MODEL TO PREDICT

HIGH FREQUENCY MODULI OF ACOUSTIC FOAMS

Xinxin Guo[1], Guqi Yan[1], Lazhar Benyahia[2], Sohbi Sahraoui[1],
[1] LAUM, CNRS UMR 6613, Université du Maine, Av. O. Messiaen, Le Mans, France
[2] IMMM, CNRS UMR 6283, Université du Maine, Av. O. Messiaen, Le Mans, France

Summary This paper presents a time domain method to determine viscoelastic properties of open-cell foams on a wide frequency range. This method is based on the adjustment of the stress-time relationship, obtained from relaxation tests on polymeric foams samples under static compression, with the four fractional derivatives Zener model. The experimental relaxation function, well described by the Mittag-Leffler function, allows to predict straightforward the frequency-dependence of complex modulus of polyurethane foams. A curve was reconstructed on the reduced frequency range (0.1Hz – 1 MHz) using the time-temperature superposition principle. Very good agreement was obtained between experimental complex moduli values and the fractional Zener model predictions. The proposed time domain method may constitute an improved alternative to resonant and non-resonant techniques often used for dynamic characterization of polymers for the determination of viscoelastic moduli on a broad frequency range.

1. Viscoelastic foam: relaxation function

Stress-strain relationship : $\sigma(t) = \int_{-\infty}^{t} R(t-\tau)\dot{\varepsilon}(\tau)d\tau$

Using the Laplace transform: $\tilde{\sigma}(p) = \tilde{R}(p)\tilde{\dot{\varepsilon}}(p)$

where: $L\{f(t);p\} = \tilde{f}(p) = \int_0^{\infty} f(t)e^{-pt}dt$

2. Fractional Zener model (FZM):

Stress-strain relation : $\left[1+a\frac{d^{\alpha}}{dt^{\alpha}}\right]\sigma(t) = \left[m+b\frac{d^{\alpha}}{dt^{\alpha}}\right]\varepsilon(t)$

Frequency domain : $\left[1+a(j\omega)^{\alpha}\right]\tilde{\sigma}(\omega) = \left[m+b(j\omega)^{\alpha}\right]\tilde{\varepsilon}(\omega)$

Complex modulus : $G^{*} = \frac{m+b(j\omega)^{\alpha}}{1+a(j\omega)^{\alpha}} = G'+jG''$

3. Time domain approach

Stress-strain relation
Laplace transform:

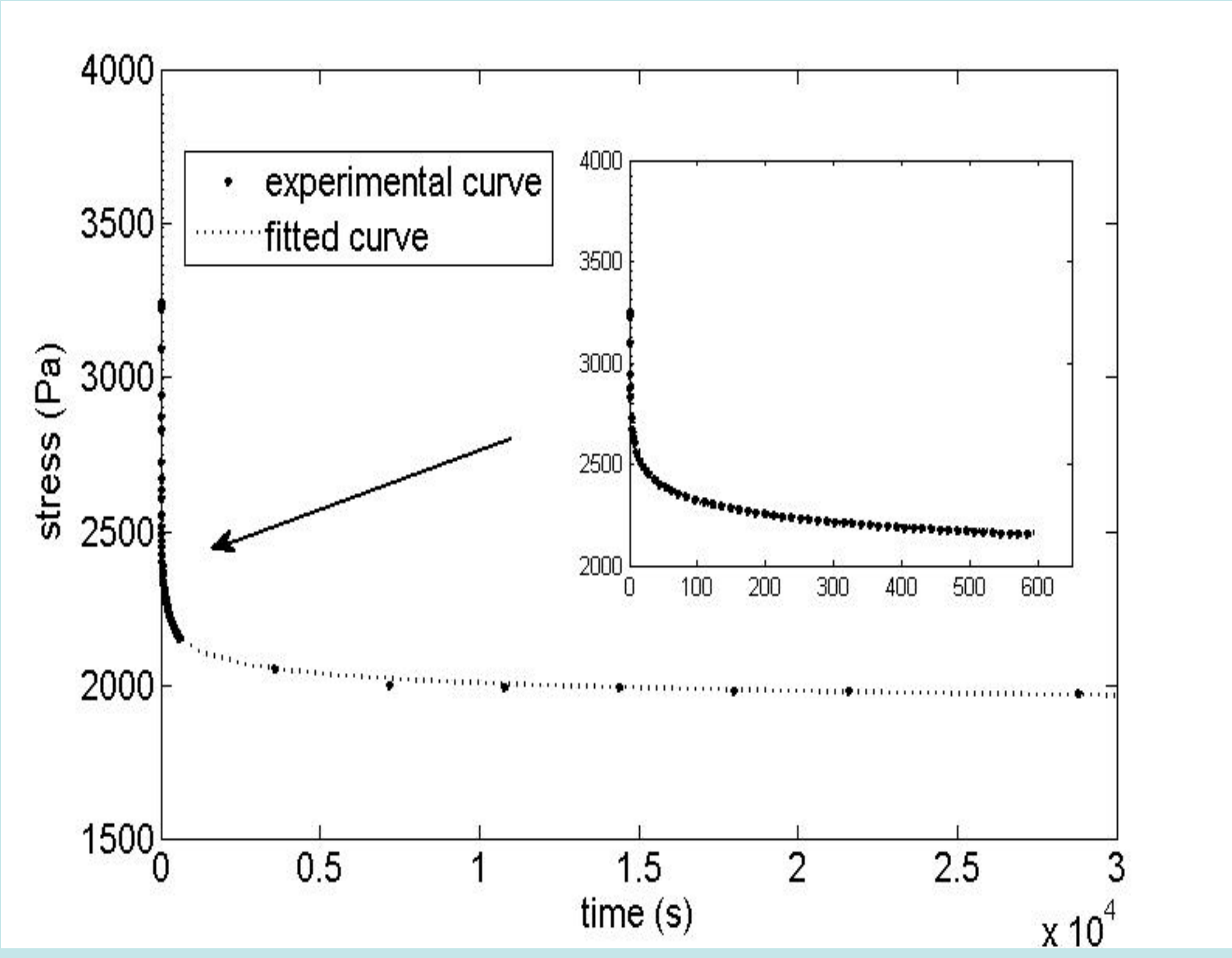
$$\tilde{R}(p) = \frac{\tilde{\sigma}(p)}{\tilde{\dot{\varepsilon}}(p)} = \frac{1}{p}\frac{m+bp^{\alpha}}{1+ap^{\alpha}}$$

$$= \frac{m}{p} + (\frac{b}{a}-m)\frac{p^{\alpha-1}}{p^{\alpha}+1/a}$$

Relaxation function: $R(t) = m + (\frac{b}{a}-m)E_{\alpha}\left[-t^{\alpha}/a\right]$

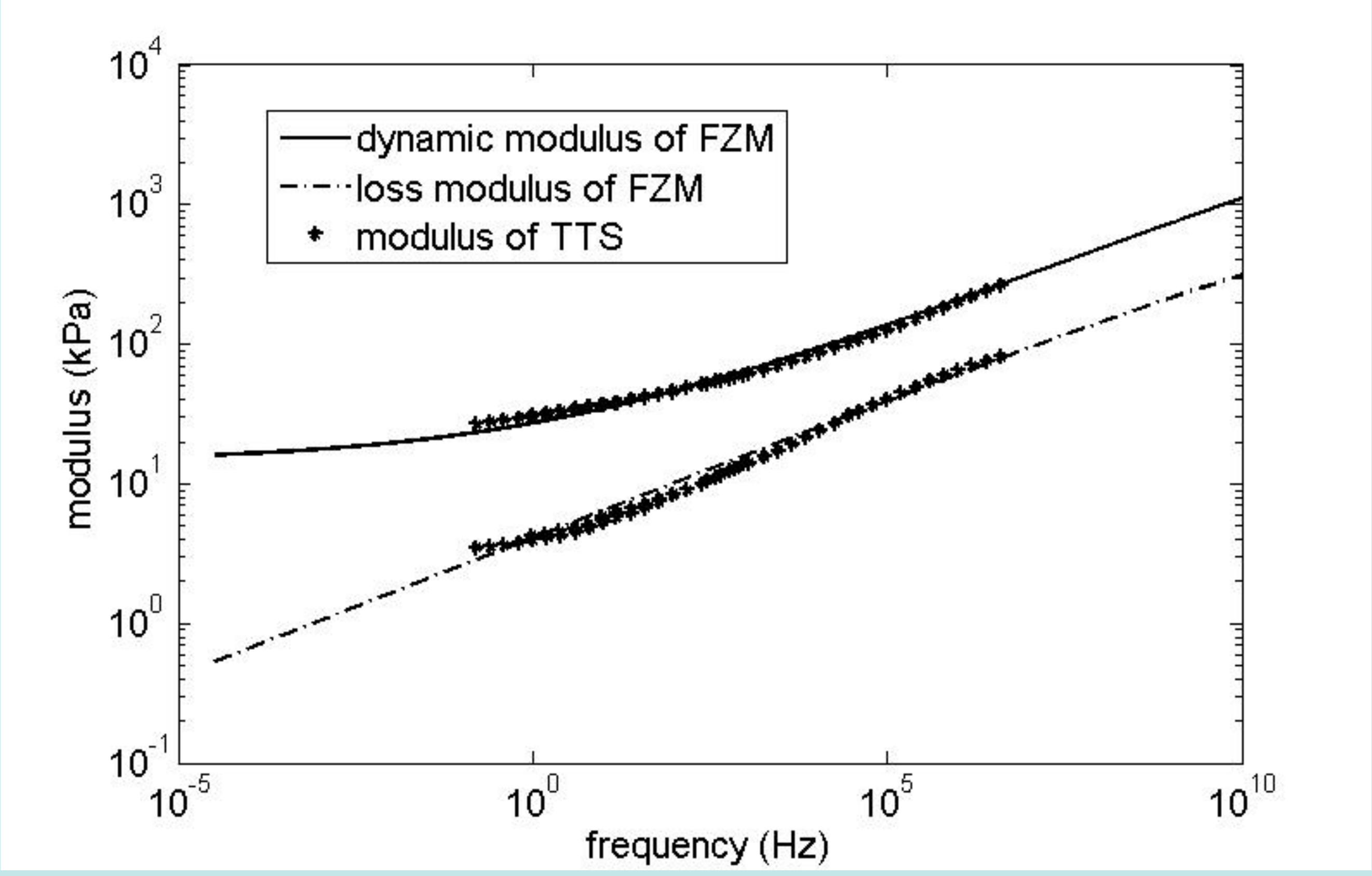
Mittag function: $E_{\alpha}\left[-t^{\alpha}/a\right] = \frac{1}{\pi}\int_0^{\infty} e^{-\tau t}\frac{\tau^{\alpha-1}\sin(\alpha\pi)/a}{1/a^2+2\tau^{\alpha}\cos(\alpha\pi)/a+\tau^{2\alpha}}d\tau$

4. Relaxation tests



Stress relaxation: experimental and FZM fitted curves. The insert is a zoom of the first 10 min

5. Predictions in wide frequency range



Storage and loss moduli comparison. Time temperature superposition experiments are not presented here

Conclusion In this work, by adjusting the FZM on the stress relaxation experiments of acoustic foams, we were able to extract the four parameters of the model. It has been shown that the relaxation function obtained in just 10 min in a single static compression test was sufficient to calculate the complex modulus at any frequency. The comparison between the experimental results and the predicted values of the FZM has shown a good agreement in a frequency range between 0.1 Hz and 10^6 Hz.

Etchessahar M., Sahraoui S., Benyahia L., Tassin J. F. : J. Acoust. Soc. Am.117 (3), 1114–1121, 2005
1.Pritz T.: J. Sound Vib. **195**, 103–115, 1996
2.Mainardi F.: Fractional calculus and waves in linear viscoelasticity. Imperial College Press, London, 2010
3.Williams, M.L., Landel, R.F., Ferry, J.D.: J. Am. Chem. Soc.**77**, 3701–3706, 1955
4.Guo X., Yan G., Benyahia L., Sahraoui S.: Mech Time-Depend Mater, , **20** (4), 523-533, (2016)

SAPEM 2017, december 6-7-8, Le Mans, France