On the use of uncertainties from the characterization to the computation of dispersion envelope of poro-elastic media

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Thanks to Stephen Hillenburg and Matthew Edwards
Outline

1 Introduction
   - Introduction
   - Overview of the used characterisation method

2 Uncertainties and standard deviations
   - Extraction of relevant information from measurements
   - Global standard deviation on JCAL parameters
   - Single average curve versus enveloppe

3 Envelope estimation with complete computation
   - Method principle
   - Computation cost

4 Sampling method
   - Method principle
   - $\beta$ influence
   - Automatic sampling method
   - Sampling method calculation cost

5 Conclusion
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Introduction

- The modelling of the acoustical behaviour of a poro-elastic material usually requires **11 parameters** (6 JCAL + 4 Elastic + thickness)
- Each parameter is measured or characterized with a given uncertainty
- Several samples per material $\rightarrow$ **standard deviations** for each parameter
- The idea of this work is to propose a **methodology** to take into account both measurement uncertainties and standard deviations between samples to simulate envelopes in place of a single average curve.
- 2 methods are investigated: the complete method and the sampling method.
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Overview of the used characterisation method

**Direct methods**
- Air flow resistivity measurement
  - ISO 9053
- Porosity measurement
  - Beranek, Salissou & Panneton

**Indirect methods: (Audible range) Analytical inversion**
- for porous materials: Olny & Panneton, from measurement of $\rho_{eq}$ and $K_{eq}$
- for perf. plates & fabrics: Jaouen & Bécot, from measurement of $Z_s$
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Static air flow resistivity

- ISO 9053
Open porosity

Beranek et al. 1942, further modified by Champoux, Stinson and Daigle 1991

Salissou & Panneton

[Diagram of a setup with labeled parts such as Valve, Air reservoir, Differential pressure transducer, Piston, Material sample, and compartments labeled $M_1$, $M_2$, $M_3$, $M_4$.]
Impedance tube setups for measurement of $\rho_{eq}$ et $K_{eq}$

- 2 load method: Utsuno et al.
- 3 mic. method: Iwase et al.
- 4 mic. method: Song & Bolton
Analytical inversion for porous materials: $\alpha_\infty$, $\Lambda$, $\Lambda'$, $k'_0$

From measurement of $\rho_{eq}$ and $K_{eq}$, separate assessment of the visco-inertial and thermal effects following the Johnson-Champoux-Allard-Lafarge model

\[
\tilde{\rho}_{eq} = \frac{\alpha_\infty \rho_0}{\phi} \left[ 1 - j \frac{\sigma \phi}{\omega \rho_0 \alpha_\infty} \sqrt{1 + j \frac{4 \alpha_\infty^2 \eta \rho_0 \omega}{\sigma^2 \Lambda^2 \phi^2}} \right],
\]

or

\[
\tilde{\rho}_{eq} = \tilde{A} + j \tilde{B}
\]

\[
\tilde{K}_{eq} = \frac{\gamma P_0 / \phi}{\gamma - (\gamma - 1) \left[ 1 - j \frac{\phi \kappa}{k'_0 C_p \rho_0 \omega} \sqrt{1 + j \frac{4 k'_0^2 C_p \rho_0 \omega}{\kappa \Lambda'^2 \phi^2}} \right]^{-1}},
\]

or

\[
\tilde{K}_{eq} = \tilde{C} + j \tilde{D}
\]

\[
\alpha_\infty = \frac{\phi}{\rho_0} \left( \text{Re}(\tilde{\rho}_{eq}) - \sqrt{[\text{Im}(\tilde{\rho}_{eq})]^2 - \frac{\sigma^2}{\omega^2}} \right)
\]

\[
\text{and}
\quad \Lambda = \frac{\alpha_\infty}{\phi} \sqrt{\frac{2 \eta \rho_0}{\omega \text{Im}(\tilde{\rho}_{eq})} \left[ \frac{\alpha_\infty \rho_0 / \phi - \text{Re}(\tilde{\rho}_{eq})}{\text{Im}(\tilde{\rho}_{eq})} \right]}
\]

\[
\quad \text{and}
\quad \Lambda' = 2 \sqrt{\frac{\kappa}{C_p \rho_0 \omega} \left\{ -\text{Im} \left( \frac{\gamma P_0 - \phi \tilde{K}_{eq}}{\gamma P_0 - \gamma \phi \tilde{K}_{eq}} \right)^2 \right\}^{-1}}
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Static error estimation from parameters’ spectra

*Mode: the value in a given dataset that appears most frequently

Viscous characteristic length (\(\mu m\)): standard deviation estimation from a **large frequency interval**

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<tr>
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</tr>
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<tr>
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<td>42</td>
<td></td>
</tr>
<tr>
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Measurement uncertainties and standard deviations between samples

- **Common method**: 
  \[ \forall i \in [1, n], \ i \text{ sample index, and } N_i \text{ the frequency number, } \]
  \[ std_m = \sqrt{\frac{\sum_i^n (\bar{x}_i - \bar{X}_m)^2}{n-1}} \text{ with } \bar{X}_m = \frac{1}{n} \sum_1^n \bar{x}_i \]
  and on each sample \( std_i = \sqrt{\frac{\sum_i^{N_i} (x_i - \bar{x}_i)^2}{N_i-1}} \)

- **Exact expression**: 
  \[ std = \sqrt{\frac{\sum_1^n \sum_i^{N_i} (x_i - \bar{X})^2}{\sum_1^n N_i-1}} \text{ with } \bar{X} = \frac{1}{\sum_i N_i} \sum_1^n \sum_i^{N_i} x_i \]

- **Proposed method**: 
  Hypothesis: \( N_1 = N_2 = \ldots = N_n = N \) and \( N \) big enough
  \[ std \approx \sqrt{\frac{\sum_1^n \sum_i^{N_i} (x_i - \bar{x}_i)^2}{n}} + std_m^2 \]
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\[ \text{julia.rodenas@matelys.com} \]
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<th>Proposed method</th>
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<td>( \Lambda_{\text{sample1}} ) = 142 ± 40( \mu m )</td>
<td>142 ± 0( \mu m )</td>
<td>142 ± 51( \mu m )</td>
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<tr>
<td>( \Lambda_{\text{sample2}} ) = 142 ± 60( \mu m )</td>
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**Comparison between the common and the proposed method**: Example for the viscous characteristic lengths of 2 samples.
Foam modeled with **JCAL parameters** given by:
- thickness = 30.00 ± 0.22 mm,
- $\phi = 0.99 \pm 0.01$,
- $\sigma = 109\ 200 \pm 41\ 000$ N.s.m$^{-4}$,
- $\Lambda = 13 \pm 5$ $\mu$m,
- $\Lambda' = 105 \pm 19$ $\mu$m,
- $\alpha_\infty = 1.13 \pm 0.13$

- A simulated envelope would be more appropriate than the simulation with mean values.
- Other indicators as absorption or transmission under a diffused field could be investigated.
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Method principle

- **N parameters** - Example for JCAL/elastic model ⇒ 11 parameters
- 2 possibilities per parameter, minimum and maximum values - Example for \( \Lambda = 13 \pm 5 \, \mu m \) ⇒ \( \Lambda_{min} = 8 \, \mu m \) and \( \Lambda_{max} = 18 \, \mu m \)
- All combinations are computed - Example for JCAL/elastic model ⇒ \( 2^{11} = 2048 \) cases to compute
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3 types of quantities:

- the sound absorption coefficient under a diffuse field excitation $\alpha_{DF}$,
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**Complete computation cost**: Computational time may be prohibitive for systematic envelope evaluation for a large number of samples.

**Computer capacities**: processor Intel(R) Core(TM) i5-4200M CPU @ 2.50GHz and 8 GB of RAM
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- For each sample, parameters are **randomly chosen in a range** defined by:
  \[ \forall P \text{ parameter} \quad | P \in [\bar{X}_{\text{param}} - \beta \times \text{std}_{\text{param}}, \bar{X}_{\text{param}} + \beta \times \text{std}_{\text{param}}] \]
  with \( \beta \) a coefficient \( \beta \in [1, 3] \)

- **Dismiss unrealistic samples** - Example: a sample cannot have a negative resistivity even if its resistivity equals 40 000 \( \pm \) 50 000 N\( \cdot \)s\( \cdot \)m\(^{-4} \).

- Experience showed that **20 samples** are enough to simulate one envelope.

- Once the set of all samples are generated, simulations are launched for each sample.

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$\beta = 1$
\( \beta \) influence

\[ P = \bar{X}_{\text{param}} \pm \beta \times \text{std}_{\text{param}} \]

\( \beta = 1 \)

\( \beta = 2 \)
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- \( \beta = 1 \)
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Automatic sampling method principle:

- An **iterative procedure** is used to find the best $\beta$,
- The algorithm is stopped when the difference between the generated population and the measured one is less than 5% for both the mean value and the standard deviation.
- The selection of samples is done before running computations so its time cost is negligible.
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• Similar envelopes are obtained using two different methods: a complete method which uses minimum and maximum values for each parameter, and a sampling method using only few representative samples.

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• This method could be used with other types of simulation.
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