Symposium on the Acoustics of Poro-Elastic Materials
Porous media, metamaterials and sonic crystals
6-7-8 December 2017, Le Mans (France)

SLaTCoW (Spatial LAplace Transform for COmplex Wavenumber recovery)

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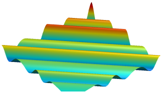
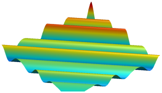
Context: dispersive and attenuating medium



- Inhomogeneous waves propagation
- Complex wavenumber $k \in \mathbb{C}$
- $\mathrm{Re}(k) \ \rightarrow$ wave dispersion
- $\mathrm{Im}(k) \ \rightarrow$ wave attenuation

Context: dispersive and attenuating medium



- Inhomogeneous waves characterization
- Complex dispersion relation $k(\omega) \in \mathbb{C}$
- Assess medium properties. Applications: material characterization, attenuation analysis, etc.
- Dispersion relation tuned by designing the material microstructure. Applications: imaging, focusing, lensing, etc.

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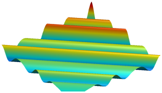

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Objective: propose a method for complex wavenumber recovery

- Robust in frequency for characterizing a complex dispersion relation.
- Robust regarding experimental configurations (noise ratio, modes overlapping, energy spreading from the source, etc).

Outline

SLaTCoW Method

Application: Porous material mechanical characterization ($0.2\ kHz$ to $4\ kHz$)

Application: ZGV Lamb mode ($1.85\ MHz$ to $2.03\ MHz$)

Application: SAW in microscale granular crystal ($50\ MHz$ to $400\ MHz$)

Application: SAW at a lossy metasurface (point source excitation)

Conclusions and perspectives

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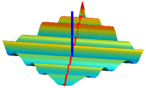
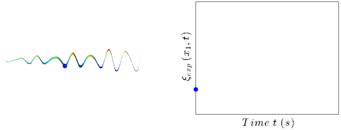
Spatial LAplace Transform for COmplex Wavenumber

(Geslain *et al.*, JAP, 2016)





- Measurement point $\xi_{\text{exp}}(x_1, t)$

Spatial LAplace Transform for COmplex Wavenumber

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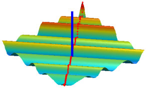

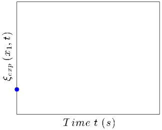

- Measurement field $\xi_{\text{exp}}(x, t)$

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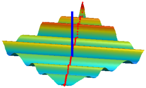

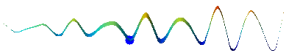

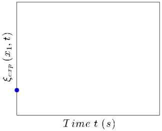

- Measurement field $\xi_{\text{exp}}(x, t)$
- Usual procedure using Time Fourier Transform (TFT) $\mathcal{F}\left[\xi_{\text{exp}}(x, t)\right] \rightarrow \xi_{\text{exp}}(x, \omega)$

Spatial LAplace Transform for COmplex Wavenumber

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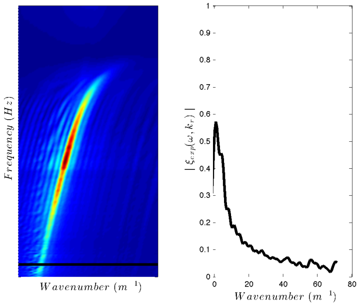

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  $\mathcal{F}[\xi_{\text{exp}}(x, t)] \rightarrow \xi_{\text{exp}}(x, \omega)$
- Usual procedure using Spatial Fourier Transform (SFT)
  $\mathcal{F}[\xi_{\text{exp}}(x, \omega)] \rightarrow \xi_{\text{exp}}(k_r, \omega)$

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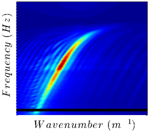

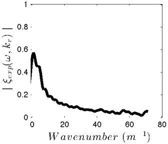

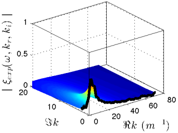

- Measurement field $\xi_{\text{exp}}\left(x,t\right)$
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  $\mathcal{F}\left[\xi_{\text{exp}}\left(x,t\right)\right] \rightarrow \ \xi_{\text{exp}}\left(x,\omega\right)$
- Usual procedure using Spatial Fourier Transform (SFT)
  $\mathcal{F}\left[\xi_{\text{exp}}\left(x,\omega\right)\right] \rightarrow \ \xi_{\text{exp}}\left(k_r,\omega\right)$
- <u>Present</u> procedure SLaTCoW using Spatial Laplace Transform (SLT)
  $\mathcal{L}\left[\xi_{\text{exp}}\left(x,\omega\right)\right] \rightarrow \ \xi_{\text{exp}}\left(k_r,\underline{k_i},\omega\right)$

Spatial LAplace Transform for COmplex Wavenumber

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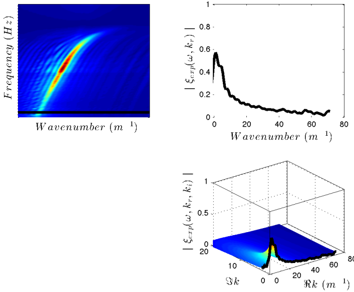

- SLaTCoW
  $\mathcal{L}\left[\xi_{\text{exp}}\left(x,\omega\right)\right] \rightarrow \ \xi_{\text{exp}}\left(k_r, k_i,\omega\right)$

- $\{\zeta\}$: set of parameters which describes theoretically the propagating field $\xi_{th}(\{\zeta\})$
- $\{\zeta\}$ retrieved by minimizing the difference between $\mathcal{L}\left[\xi_{\text{exp}}\right]$ and $\mathcal{L}\left[\xi_{\text{th}}\right]$

$$\{\zeta\} = \text{argmin}\, \Delta(\{\zeta'\}) \quad ; \quad \Delta(\{\zeta'\}) = \sqrt{\sum_s \left|\mathcal{L}\left[\xi_{\text{exp}}\right](s) - \mathcal{L}\left[\xi_{\text{th}}\left(\{\zeta'\}\right)\right](s)\right|^2}$$

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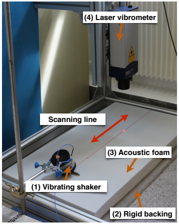
Conclusions and perspectives

Guided waves in poroelastic material: set-up



- Acoustic foam (85 *cm*, 45 *cm*, 5.5 *cm*)
- Excitation system (shaker + « T » shaped termination)
- 300 sine functions 200 *Hz* 4095 *Hz*
- Scanning line L = 40 *cm*
- Normal displacement $\xi_{exp}(x, \omega) = u_n(x, \omega)$

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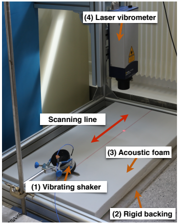

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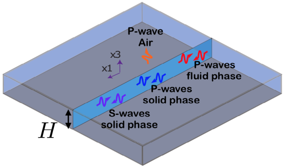
Retrieving $\{\zeta\} = \{A_n, \phi_n, k_r^n, k_i^n\}$ from Ansatz plane waves

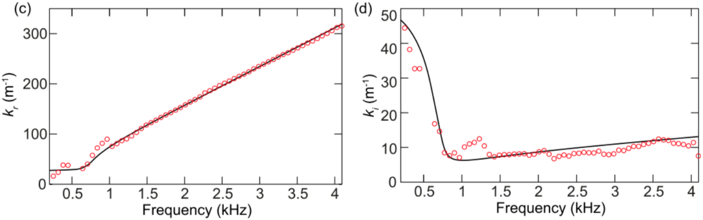
- $\xi_{th}(\{\zeta'\}, x, \omega) = \sum_n A_n e^{i\phi_n} e^{(ik_r^n - k_i^n)x} * \Pi(x-L)$ with $n = 3$ waves
- Minimization $\{\zeta\} = \text{argmin}\,\Delta(\{\zeta'\})$ with

$$\Delta(\{\zeta'\}) = \sqrt{\sum_s \left|\mathcal{L}\left[\xi_{\text{exp}}(x,\omega)\right](s) - \mathcal{L}\left[\xi_{\text{th}}\left(\{\zeta'\}, x, \omega\right)\right](s)\right|^2}$$

Guided waves in poroelastic material



- Biot 62 formulation $\{u, w\}$
- 7 waves amplitudes problem $AX = 0$
- Complex matrix determinant resolution $det(A)$ to retrieve k_r and k_i



ϕ	$\rho\ (kgm^{-3})$	α_∞	$\sigma(Nsm^{-4})$	$\Lambda(\mu m)$	$\Lambda'(\mu m)$	$N(KPa)$	ν
0.9	6.1	1	8060	215	215	$38 - i1.52$	0.3

Outline

ZGV mode in duralumin plate loaded by heavy fluid

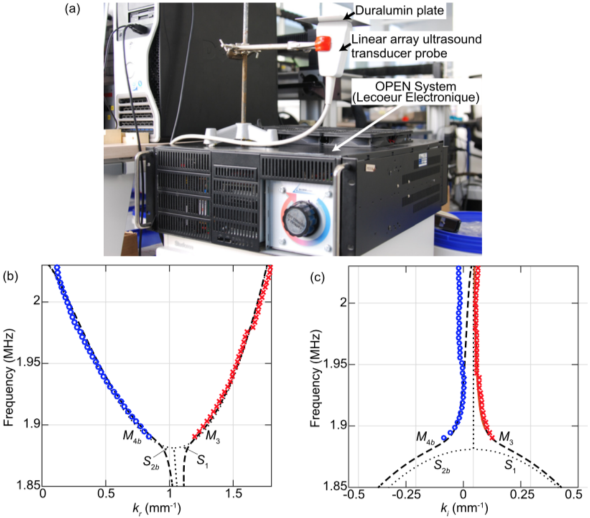
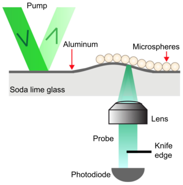


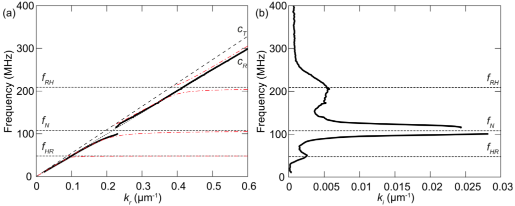
FIG. 3. (a) Photograph of the experimental setup. (b) and (c) Dispersion of Lamb waves in a 1.525 mm thick Duralumin plate: (b) real part and (c) imaginary part of the wavenumber. Markers denote the points identified using the SLaTCoW method. The chosen ranges, $k_r \in [0; 1.8]$ mm^{-1}, $k_i \in [-0.5; 0.5]$ mm^{-1}, and $f \in [1.85; 2.03]$ MHz, include two Lamb modes: M^3 (cross) and M^{4b} (circle). Theoretical complex dispersion curves using two different models are displayed for comparison: a model considering a free-standing Duralumin plate (dotted lines) and a more complex model (dashed lines).

- Leakage leads to mode repulsion

Outline

SAWs propagating in an adhered microsphere monolayer



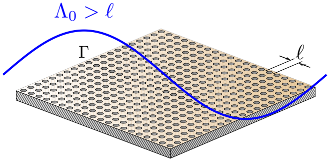


- Dispersion curves (solid lines SLaTCoW method, red dash-dotted lines lossless model)
- f_N out of-plane contact resonance (Rayleigh waves propagation)

Outline

SAW at lossy metasurface (Spoof)

(Schwan *et al.*, APL, 2017)



Wood surface with periodic boreholes

- Square lattice $\ell = 5$ *cm*
- Circular resonator (radius $a = 1.8$ *cm*)

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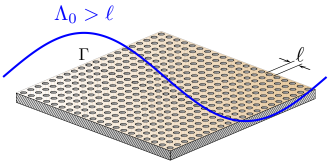

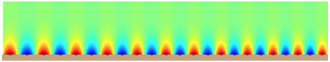

- Acoustic pressure (propagating field & boundary layer perturbation)

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Plane wave above the metasurface

- SAW wavenumber k complex

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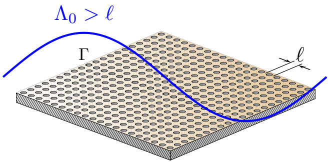

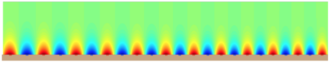

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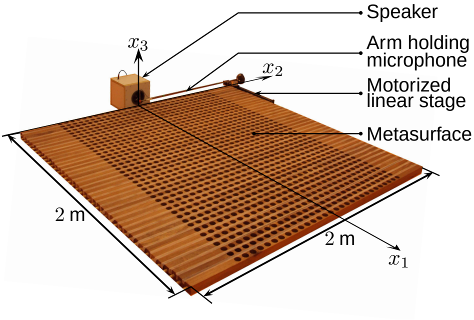

Ansatz for a point source excitation

Point source above an admittance surface

(Wenzel, JASA, 1974) (Chien et al., JSV, 1975)

$$p_{\text{th}}(\mathbf{x}) = A_0\mathcal{G}_0 + A_s\mathcal{G}_s$$

$$\mathcal{G}_0 = \frac{e^{\mathrm{i}k_0R_1}}{4\pi R_1} + \frac{e^{\mathrm{i}k_0R_2}}{4\pi R_2}$$

$$\mathcal{G}_s = -k_0\mathrm{erfc}\left(-\mathrm{i}\mathrm{w}\right)\mathrm{H}_0^{(1)}(kr)e^{-b(x_3+z_s)}/4$$

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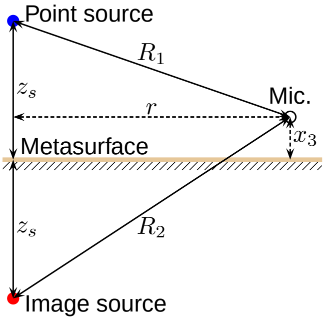

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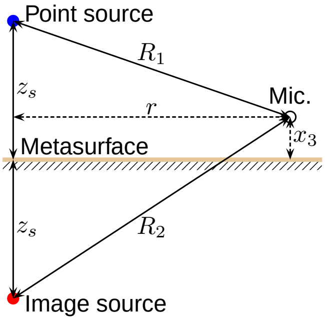

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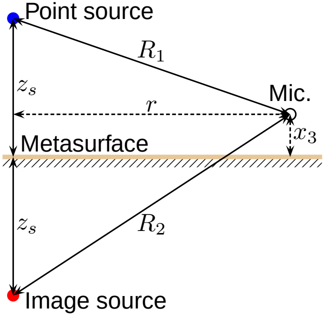

- Geometrical spreading of the field from the source
- Parameters describing the field:
  $\{\zeta\} = \{\mathrm{Re}(k_0), \mathrm{Im}(k_0), \mathrm{Re}(k), \mathrm{Im}(k), |A_0|, \arg(A_0), |A_s|, \arg(A_s)\}$

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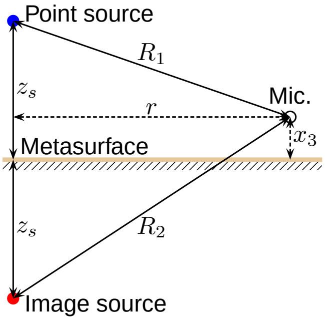

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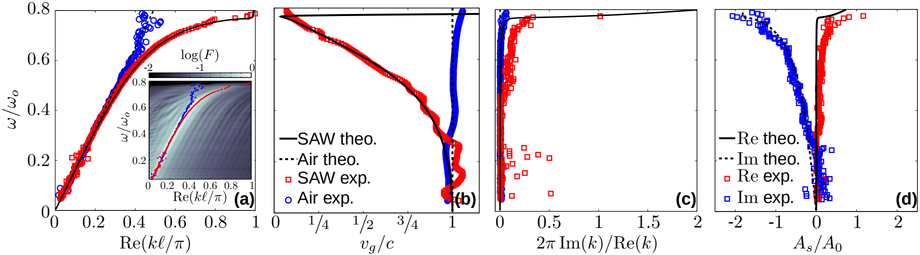
- Theoretically: $A_s/A_0 = \rho_e c\Upsilon$

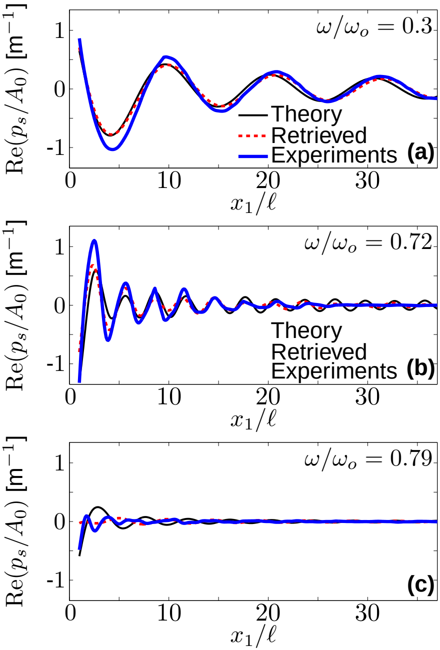
SAW complex dispersion relation



- Low frequency $\omega \ll \omega$: negligible SAW amplitude.
- Approaching resonance $\omega \to \omega_o$: Evidence of surface wave $\mathrm{Re}(k)(\omega) \geq k_0$ and slow sound $v_g/c \sim 0.2$ with increasing attenuation.
- Approaching Bragg limit $\mathrm{Re}(k\ell) \to \pi$: Near zero v_g/c but very high attenuation $1/\mathrm{Im}(k)(\omega) \leq 2\pi/\mathrm{Re}(k)(\omega)$.

Pressure profiles

- Pressure field more and more confined near the source as $\omega \to \omega_0$.



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Conclusions and perspectives

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SLaTCoW is an efficient, robust, and versatile method:

- to measure the complex dispersion relation for a large frequency range
- to account to several modes, which may overlap
- to account for geometrical spreading with an appropriate ansatz
- limitations: choose an appropriate ansatz
  & have enough noise ratio

Perspectives

- poroelasticity characterization with refine viscoelastic model
- 2D dispersion relation/resonant sonic crystals (Lecture of A. Cebrecos at DENORMS Training School)
- 3D dispersion relation
- Application to other domain of physics

Thank you for your attention

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Rendez-vous au Havre (23-27 Avril 2018)
14ème Congrès Français d'Acoustique
sessions spécialisées :
Matériaux poreux
Matériaux structurés pour l'acoustique et les vibrations