Acoustic Response of Anisotropic Multilayered Structures

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Motivation

- Open-cell porous materials are inherently anisotropic due to manufacturing processes, comprising a highly irregular microscopic structure.
- The anisotropy of porous media presents a potential for the optimal design of multilayered systems for multifunctional performance.
- There is a need for modeling tools to study the influence of the inherent mechanical and acoustic anisotropy of porous materials on the acoustic behavior of anisotropic multilayered structures.

Objectives

- A plane wave method is presented to study the influence of material orientation on the dynamic behavior of multilayered panels including anisotropic porous media.
- A state vector approach is used and the solution is obtained recursively as a superposition of waves in each layer[1].
- This allows to obtain the acoustic and mechanical solutions of anisotropic multilayered systems under single-sided excitation with arbitrary angle of incidence.
- Performance indicators of the simplified system and its composing layers derive therefrom, such as powers dissipated by each individual material layer.
- The model has been extended to include the computation of the acoustic response of multilayered systems under an acoustic direct field excitation.

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Problem: Dynamic behaviour of an homogeneous layer

The state-space representation requires the computation of the state vector at a known point How to compute the state matrix? 

Solution: Plane wave approach

Every variable may be expressed in the form: $\mathbf{x}(t) = \mathbf{v} e^{i \lambda t}$. Thus, the behavior of the layer can be written as with the transfer matrix and the state vector at a known point $t_0$, which can be simplified as $A_e \mathbf{w}(t_0) = \mathbf{w}(t_0)$ where $\mathbf{A}_e$ is the state matrix.

Method: State Matrix

The generalised an-symmetric eigenvalue problem associated to $R$ and $A_e$ can be written as $A_R \mathbf{w}(t_0) = \mathbf{w}(t_0)$. Due to the rank deficiency of the problem, the eigenvectors and eigenvalues may be separated into two sets: $A_R \phi = \lambda \phi$, $\mathbf{V} \Phi \mathbf{R}^{-1} \Phi^T \mathbf{V}^T = \mathbf{O}$. The system of equations describing the state of the layer may be then expressed as $R_e (\mathbf{L}_e - \alpha \mathbf{A}_R \Phi \mathbf{R}^{-1} \Phi^T \mathbf{L}_e - \alpha) \mathbf{w}(t_0) = \mathbf{w}(t_0)$. Where $\mathbf{w}(t_0)$ is the length of $\mathbf{w}(t)$ and $R = \mathbf{V} \Phi \mathbf{R}^{-1}$. By partitioning the latter into $R_e (\mathbf{L}_e - \alpha \mathbf{A}_R \Phi \mathbf{R}^{-1} \Phi^T \mathbf{L}_e - \alpha) \mathbf{w}(t_0)$ a direct relation between the state vector and the remaining variables may be established as $\mathbf{w}(t_0) = \mathbf{P}(\lambda) \mathbf{w}(t_0)$, where $\mathbf{P}(\lambda)$ is the state matrix for a linear homogeneous generally anisotropic media as $\alpha = (\Phi A \Phi^{-1}) T$. The state matrix for a linear homogeneous generally anisotropic media as $\alpha = (\Phi A \Phi^{-1}) T$. 

Waves in the anisotropic poroelastic core

- The core of the studied system is composed of an anisotropic poroelastic homogeneous multilayered core.
- The flow resistivity and the stiffness of the material has been measured and inserted as fully anisotropic.
- The wave behavior is the stiffness of the core in the wave propagation through the medium.
- The alignment of a multilayered core is arbitrary, and shapes always a degree of shear/compression coupling.

Core parameters

- Core: $\mathbf{P}(\lambda)$
- Core: $\mathbf{A}_e$
- Core: $\mathbf{R}_e$
- Core: $\mathbf{L}_e$
- Core: $\mathbf{V}$
- Core: $\Phi$
- Core: $\mathbf{R}$
- Core: $\mathbf{L}$
- Core: $\mathbf{A}$
- Core: $\mathbf{B}$
- Core: $\mathbf{C}$
- Core: $\mathbf{D}$
- Core: $\mathbf{E}$
- Core: $\mathbf{F}$
- Core: $\mathbf{G}$
- Core: $\mathbf{H}$
- Core: $\mathbf{I}$
- Core: $\mathbf{J}$
- Core: $\mathbf{K}$
- Core: $\mathbf{L}$
- Core: $\mathbf{M}$
- Core: $\mathbf{N}$
- Core: $\mathbf{O}$
- Core: $\mathbf{P}$
- Core: $\mathbf{Q}$
- Core: $\mathbf{R}$
- Core: $\mathbf{S}$
- Core: $\mathbf{T}$
- Core: $\mathbf{U}$
- Core: $\mathbf{V}$
- Core: $\mathbf{W}$
- Core: $\mathbf{X}$
- Core: $\mathbf{Y}$
- Core: $\mathbf{Z}$
- Core: $\mathbf{a}$
- Core: $\mathbf{b}$
- Core: $\mathbf{c}$
- Core: $\mathbf{d}$
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- Core: $\mathbf{o}$
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- Core: $\mathbf{q}$
- Core: $\mathbf{r}$
- Core: $\mathbf{s}$
- Core: $\mathbf{t}$
- Core: $\mathbf{u}$
- Core: $\mathbf{v}$
- Core: $\mathbf{w}$
- Core: $\mathbf{x}$
- Core: $\mathbf{y}$
- Core: $\mathbf{z}$

Sub-layering of the core

- The core is divided into 10 sub-layers of same thickness (10 mm).
- Each sub-layer has the same nominal characteristic, and a multilayer coordinate system independent from one another.
- 6 different material coordinate system rotation profiles are studied. In each case, the multilayer coordinate system of each sub-layer is rotated. The sub-layered coordinate system rotation profiles may be seen in Fig. 5. The profiles which choose to study the influence of relative rotation of the material coordinate system of the sub-layers.

Sub-layered core

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Core</th>
<th>Sublayer 1</th>
<th>Sublayer 2</th>
<th>Sublayer 3</th>
<th>Sublayer 4</th>
<th>Sublayer 5</th>
<th>Sublayer 6</th>
<th>Sublayer 7</th>
<th>Sublayer 8</th>
<th>Sublayer 9</th>
<th>Sublayer 10</th>
</tr>
</thead>
<tbody>
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<td>200</td>
<td>Core</td>
<td>Sublayer 1</td>
<td>Sublayer 2</td>
<td>Sublayer 3</td>
<td>Sublayer 4</td>
<td>Sublayer 5</td>
<td>Sublayer 6</td>
<td>Sublayer 7</td>
<td>Sublayer 8</td>
<td>Sublayer 9</td>
<td>Sublayer 10</td>
</tr>
<tr>
<td>400</td>
<td>Core</td>
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<td>Sublayer 2</td>
<td>Sublayer 3</td>
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<td>Sublayer 5</td>
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<td>Sublayer 9</td>
<td>Sublayer 10</td>
</tr>
<tr>
<td>600</td>
<td>Core</td>
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<td>Sublayer 3</td>
<td>Sublayer 4</td>
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<td>Sublayer 7</td>
<td>Sublayer 8</td>
<td>Sublayer 9</td>
<td>Sublayer 10</td>
</tr>
</tbody>
</table>

Conclusions

- The proposed method for the study of multilayered systems is based on the expansion of the dynamic solutions as a superposition of plane waves.
- The formulation relies on a state-space representation in terms of physical field variables, and directly provides the characteristics of the waves in the different layers of the structure.
- The state-space representation requires the computation of the state matrix, characterizing the dynamic state of each material layer in a system.
- Using the proposed method, it was found that, for an anisotropic industrial melamine foam, there is a strong influence of the inherent anisotropy of the material in the wave propagation through the medium.

References