Properties of bulk reacting absorbers subject to flow and temperature boundary layers

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Background

Automotive applications

Noise encapsulations

$50 \leq f \leq 5000$ Hz

Grazing flow Mach no. $< 0.2$
Background

Complex sound and flow field
Noise encapsulation

- Low Mach numbers
- Bulk reacting liners
- Strong temperature gradients
- Surfaces close to source
Problem
Generic case – plane wave incidence
Bulk reaction

Local reaction
Absorber modelling – bulk reaction

Equivalent fluid model
- Transfer Matrix Method, TMM

\[
\begin{bmatrix}
  p(d) \\
  v(d)
\end{bmatrix}
= T
\begin{bmatrix}
  p(0) \\
  v(0)
\end{bmatrix}
\]

Rigid backing, \( v_0 = 0 \)

\[
Z_s = \frac{T(1,1)}{T(2,1)}
\]

\[
T_{tot} = \prod_{i=1}^{N} T_i
\]
Boundary layer modelling

Discretizing the boundary layer with TMM

\[ T_{tot} = \prod_{i=1}^{N} T_i \]
Boundary layer modelling

Discretization with gradients
• Extension of original TMM

\[ M(z) \]

\[ \frac{dM(z_n)}{dz} \]

\[ T_{tot} = \prod_{i=1}^{N} T_i \]
Flow boundary layer

- Extended Transfer Matrix Method

\[ \frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial x} + \rho_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \right) = 0 \]

\[ \frac{\partial u}{\partial t} + V \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial z} + \frac{1}{\rho_0} \frac{\partial p}{\partial x} = 0 \]

\[ \frac{\partial v}{\partial t} + V \frac{\partial v}{\partial x} + \frac{1}{\rho_0} \frac{\partial p}{\partial z} = 0 \]

\[ \frac{\partial}{\partial z} [S] = -[A][S] \]

\[ [S] = \begin{bmatrix} p \\ v \end{bmatrix} \]

\[ [T] = \expm([A]d) \]

\[ [A] = \begin{bmatrix} 0 & -i\rho(Vk_x - \omega) \\ -i \frac{c_0^2 k_x^2 - (\omega - V k_x)^2}{\rho c_0^2 (\omega - V k_x)} & k_x \frac{\partial V}{\partial z} \end{bmatrix} \]
Flow boundary layer

- Extended Transfer Matrix Method

\[
\frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial x} + \rho_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \right) = 0
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\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial x} + v \frac{\partial V}{\partial z} + \frac{1}{\rho_0} \frac{\partial p}{\partial x} = 0
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[S] = \begin{bmatrix} p \\ v \end{bmatrix}
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[A] = \begin{bmatrix}
0 & -i\rho(Vk_x - \omega) \\
-i\frac{c_0^2k_x^2 - (\omega - Vk_x)^2}{\rho c_0^2(\omega - Vk_x)} & \frac{k_x}{\omega - Vk_x} \frac{\partial V}{\partial z}
\end{bmatrix}
\]
Flow boundary layer

- Extended Transfer Matrix Method

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial x} + \rho_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \right) &= 0 \\
\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial z} + \frac{1}{\rho_0} \frac{\partial p}{\partial x} &= 0 \\
\frac{\partial v}{\partial t} + V \frac{\partial v}{\partial x} + \frac{1}{\rho_0} \frac{\partial p}{\partial z} &= 0
\end{align*}
\]

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\frac{\partial}{\partial z} [S] = -[A][S]
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[S] = \begin{bmatrix} p \\ v \end{bmatrix}
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[T] = \text{expm}([A]d)
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[A] = \begin{bmatrix}
0 & -i \rho(Vk_x - \omega) \\
\frac{c_0^2 k_x^2 - (\omega - V k_x)^2}{\rho c_0^2 (\omega - V k_x)} & k_x \frac{\partial V}{\partial z}
\end{bmatrix}
\]
Flow field – absorber

Absorber modelled as equivalent fluid
Analysed with Transfer matrix method
Boundary layer discretized
Each sub-layer calculated with the eTMM
Flow field, no flow and $M = 0.15$
- **local** and **bulk** reacting liner

Local and bulk reaction

$f = 3$ kHz $\quad \sigma = 5$ k Rayl/m
$\delta = 2$ cm $\quad d = 2$ cm
Flow field, no flow and $M = 0.15$ - local and bulk reacting liner

Local and bulk reaction

$f = 3 \text{ kHz}$ \quad $\sigma = 5 \text{ k Rayl/m}$

$\delta = 2 \text{ cm}$ \quad $d = 2 \text{ cm}$
Flow field
- impact of increasing Mach number

Bulk reaction

\[ f = 3 \text{ kHz} \quad \sigma = 5 \text{ k Rayl/m} \]
\[ \delta = 2 \text{ cm} \quad d = 2 \text{ cm} \]
Flow field - impact of increasing Mach number

Bulk reaction

\[ f = 3 \text{ kHz} \quad \sigma = 5 \text{ k Rayl/m} \]

\[ \delta = 2 \text{ cm} \quad d = 2 \text{ cm} \]
Temperature gradient and flow boundary layer
Diffuse field absorption – relative change

$M_{max} = 0.05 - 0.1$ \hspace{1cm} $\sigma = 5 - 10$ kRayls/m

$T_{max} = 100 - 150$ C \hspace{1cm} boundary layer thickness 2 cm
Internal flow

Modified flow resistivity:

\[ \sigma_v = \sigma_0 + 2\sigma_i |V_a| \] parallel to sound propagation

\[ \sigma_n = \sigma_0 + \sigma_i |V_a| \] perpendicular to sound propagation

Flow profiles:

a) Incident sound wave \( V_a \)

b) Incident sound wave \( V_a \)
Absorption coefficient, normal incidence

$\sigma_0 = 5430 \text{ Ns/m}^4$
$\sigma_i = 3070 \text{ Ns}^2/\text{m}^5$
$V_a = 0 \text{ m/s}$
$V_a = 0.415 \text{ m/s}$
$V_a = 0.715 \text{ m/s}$

19.8 mm
Summary and conclusions

Effective surface impedance with transfer matrix method

Influence of flow, temperature and internal flow

Significant effects for bulk reacting absorber

The performance of the absorber depends strongly on the specific acoustic field
Acoustic absorption

- Absorption coefficient

\[ \alpha(\theta) = 1 - \frac{W_{re}}{W_{in}} \]

- Surface impedance

\[ Z_s(\theta) = \frac{p}{v_n} = \frac{\cos \theta}{\rho_0 c_0} \frac{1 + R(\theta)}{1 - R(\theta)} \]
Flow boundary layer
- transfer matrix method (TMM)

Reference solution:
Numerical solution of Pridmore-Brown

\[ \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = (1 - M^2) \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} - 2 \frac{M}{c} \frac{\partial^2 p}{\partial x \partial t} + 2 \rho_0 c \frac{dM}{dz} \frac{\partial v}{\partial x} \]

Discretization of boundary layer
- TMM for each sub-layer in the boundary layer
Verification
Determination of absorption and impedance

Absorbed energy varies with the sound field
Sound field

Standardized measurement methods (idealized sound fields)

- Normal incidence
- Diffuse field
Sound field

\[ \sigma = 8 \text{ k Rayl/m} \]
\[ d = 2 \text{ cm} \]
Overview

Pass-by noise test simulation

Transfer matrix model for absorber and medium above

Effects of flow, temperature and internal flow on absorber performance

Absorption performance is a global property: Depends on both material and sound field incident on absorber/lining
Impact on system level

Noise requirements on pass-by noise levels.
Pass-by simulation procedure in prototyping

1. Determination of the different sources.

2. Determination of the transfer function between each source and receiver position.

3. Construction of a time signal from each transfer function.

4. Summation of noise contribution from several sources to a total SPL(A).
Simulation of pass-by noise test

Simplified model of a truck
Simulation of pass-by noise

Simplified model of a truck
Simulation of pass-by noise

Simplified model of a truck

Panels
Wheel
Beam
Engine +gearbox
Simulation of pass-by noise

Simplified model of a truck

- Bulk reacting material (reference)
- Locally reacting surface
- Extended local reaction
Simulation of pass-by noise

Simplified model of a truck

- Bulk reacting material (reference)
- Locally reacting surface
- Extended local reaction

Compare results

$$\Delta L_p = L_{p,X} - L_{p,bulk}$$
Problem
Bulk reacting

Locally reacting, $Z_s$
Extended local reaction
Simulation of pass-by noise

Simplified model of a truck

- Bulk reacting material (reference)
- Locally reacting surface
- Extended local reaction

Compare results

$$\Delta L_p = L_{p,X} - L_{p,bulk}$$
Impact of absorber model (local, extended)

\[ \Delta L_p = L_{p,X} - L_{p,bulk} \]
Impact of absorber model - different flow resistivity

\[ \Delta L_p = L_{p,X} - L_{p,bulk} \]
Impact of absorber model

\[ \Delta L_p = L_{p,X} - L_{p,\text{bulk}} \]
Difference in sound power (local, extended)
Model verification

![Graph showing model verification](image)

- $V_a = 0$
- $V_a = 0.818$
- $V_a = 1.956$

incident sound wave

$V_a$
Layers with different flow resistivity

Transfer Matrix Method, TMM

\[ T_{tot} = \prod_{i=1}^{N} T_i \]

\[ k_x = \text{constant} \]
Summary global effects

- Pass-by simulated with FEM and different models of absorber

- Pass-by levels altered up to 1 and 2 dB between material representations.