Reflection and transmission by a double porosity layer obeying Berryman-Wang theory
PLAN

Main objectives

Double porosity model

Transmission characteristics for a layer of Berea Sandstone

Dispersion curves for Berea sandstone

Evaluation of the clogging by ultrasonics testing

Conclusion & prospects
Main objectives

- Characterization of double porosity layers by acoustic methods
  - Accessibility to double porosity parameters (porosities, permeabilities, bulk modulus…)
  - Evaluation of the clogging/pollution phenomenon due to the deposition of fine suspended particles resulting from a water flow caused by rainwater infiltration.
  - Estimation of erosion.
**Double porosity model**

- **Berea Sandstone** *(confirmed)*
  - Microporosity (Matrix porosity)
    - $\nu^{(1)} \phi^{(1)}$
    - $k_{22} > k_{11}$
    - $\nu^{(1)}\phi^{(1)} > \nu^{(2)}\phi^{(2)}$
  - Macroporosity (fractures porosity)

**Views on Electronic scanning microscope**

Distribution profile of the matrix porosity in a grain (porosimeter)

Hypothesis of double porosity medium (Berryman and Wang (Int. J. Rock Min. (2000)))

- Extension of Biot’s theory to account for both pores and fractures (phenomenological)

- Continuous medium (low frequency waves considered)

- Isotropy (randomly oriented fractures, no preferred axis for fluid flow)

- Fluid in matrix and fractures is the same but the two fluid regions may be in different states of average stress (distinguished by their superscripts)

Consequence of this model: the increased number of independent coefficients describing the medium (inertial, drag and stress-strain) with respect to Biot's initial theory.


Match with Berryman and Wang theory at low frequency but more complicated to implement
Constitutive equations for isotropic double porosity poroelastic medium

- Stress tensor and fluid pressures

\[
\sigma_{\alpha\beta} = \left(H - 2\mu\right)\varepsilon_{\delta\delta}\delta_{\alpha\beta} + 2\mu\varepsilon_{\alpha\beta} - \left[C^{(1)}\xi^{(1)} + C^{(2)}\xi^{(2)}\right]\delta_{\alpha\beta}
\]

\[
P^{(1)} = -C_{12}e + C_{22}\xi^{(1)} + C_{23}\xi^{(2)}
\]

\[
P^{(2)} = -C_{13}e + C_{23}\xi^{(1)} + C_{33}\xi^{(2)}
\]

- Relative fluid-solid displacements

\[
w^{(1)} = \nu^{(1)}\phi^{(1)}\left(U^{(1)} - u\right)
\]

\[
w^{(2)} = \nu^{(2)}\phi^{(2)}\left(U^{(2)} - u\right)
\]

\(H, C^{(1)}, C^{(2)}\) : moduli of the dPP medium depending on \(C_{ij}\)

\(C_{ij}\) : related to the material properties and to the generalized poroelastic expansion and storage coefficients

\(u, U^{(1)}, U^{(2)}\) : the displacement vectors of the solid frame, the micropore fluid and the macropore fluid.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols and units</th>
<th>Berea sandstone (from BW Int. J. Rock Min. 2000)</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of grains</td>
<td>$\rho_s$ (kg/m$^3$)</td>
<td>2600</td>
<td></td>
</tr>
<tr>
<td>Unjacketed bulk modulus for the whole</td>
<td>$K_s$ (GPa)</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>Unjacketed bulk modulus for the matrix</td>
<td>$K_s^{(1)}$ (GPa)</td>
<td>38.306</td>
<td></td>
</tr>
<tr>
<td>Total porosity</td>
<td>$\phi$</td>
<td>0.1926</td>
<td></td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>$\phi^{(1)}$</td>
<td>0.178</td>
<td></td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>$\phi^{(2)}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>$k^{(11)}$ (m$^2$)</td>
<td>$10^{-16}$</td>
<td></td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>$k^{(22)}$ (m$^2$)</td>
<td>$10^{-12}$</td>
<td></td>
</tr>
<tr>
<td>Volume fraction occupied by matrix</td>
<td>$\nu^{(1)}$</td>
<td>0.9822</td>
<td></td>
</tr>
<tr>
<td>Volume fraction occupied by fractures</td>
<td>$\nu^{(2)}$</td>
<td>0.0178</td>
<td></td>
</tr>
<tr>
<td>Biot-Willis parameter for the whole</td>
<td>$\alpha$</td>
<td>0.8462</td>
<td></td>
</tr>
<tr>
<td>Biot-Willis parameter for the matrix</td>
<td>$\alpha^{(1)}$</td>
<td>0.7389</td>
<td></td>
</tr>
<tr>
<td>Biot-Willis parameter for the fractures</td>
<td>$\alpha^{(2)}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Overall tortuosity</td>
<td>$\alpha$</td>
<td>2.9460</td>
<td></td>
</tr>
<tr>
<td>Matrix tortuosity</td>
<td>$\alpha^{(1)}$</td>
<td>3.3090</td>
<td></td>
</tr>
<tr>
<td>Fracture tortuosity</td>
<td>$\alpha^{(2)}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bulk modulus of undrained porous frame</td>
<td>$K_u$ (GPa)</td>
<td>15.2</td>
<td></td>
</tr>
<tr>
<td>Jacketed bulk modulus of matrix</td>
<td>$K_1$ (GPa)</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Jacketed bulk modulus of fractures</td>
<td>$K_2$ (GPa)</td>
<td>0.108</td>
<td></td>
</tr>
<tr>
<td>Jacketed bulk modulus of porous frame</td>
<td>$K$ (GPa)</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Shear modulus for drained medium</td>
<td>$\mu$ (GPa)</td>
<td>5.478</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_f$ (kg/m$^3$)</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>Pore fluid bulk modulus</td>
<td>$K_f$ (GPa)</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\eta$ (Pa s)</td>
<td>$10^{-3}$</td>
<td></td>
</tr>
</tbody>
</table>
Analytical model for Berea sandstone

1, 2, 3: dilatational waves

t: shear wave
Basic equations of Berryman-Wang theory

**Boundary conditions** (filtration occurs across the interfaces at $z = \pm d/2$)

\begin{align*}
    u_{0z} &= u_z + w_z^{(1)} + w_z^{(2)} & \text{Continuity of fluid volume} \\
    P_0 &= P^{(1)} & \text{Continuity of fluid pressure for phase 1} \\
    P_0 &= P^{(2)} & \text{Continuity of fluid pressure for phase 2} \\
    -P_0 &= \sigma_{zz} & \text{Continuity of normal stress} \\
    0 &= \sigma_{xz} & \text{Disappearance of tangential stress}
\end{align*}

**Reflection and transmission coefficients**

\[
\begin{bmatrix} R \\ T \end{bmatrix} = \frac{1}{2} \left( \sum_{\text{cyc}} (C_{S1} + i\tau_1) X_{S23}^+ \right) + \sum_{\text{cyc}} \left( C_{A1} - i\tau_1 \right) X_{A23}^- - \sum_{\text{cyc}} \left( C_{A1} + i\tau_1 \right) X_{A23}^+ \\
\sum_{\text{cyc}} (C_{S1} + i\tau_1) X_{S23}^+ = (C_{S1} + i\tau_1) X_{S23}^+ + (C_{S2} + i\tau_2) X_{S31}^+ + (C_{S3} + i\tau_3) X_{S12}^+
\]

\[
X_{mn}^\pm = \begin{bmatrix} C_{Fm}^{(1)} \pm i\tau_m & C_{Fn}^{(1)} \pm i\tau_n \\ C_{Fm}^{(2)} \pm i\tau_m & C_{Fn}^{(2)} \pm i\tau_n \end{bmatrix} \quad \bullet = S \text{ or } A \\
(1), (2) : \text{fluid phases}
\]
They account for the squirting out and the suction of the saturating fluid under the effect of the compressional and shear deformations of the elastic frame.

They account for the deformations of the elastic frame and express the coupling of each of the dilatational wave with the shear wave.

It depends on several parameters constituting the medium as the density, and also on the acoustical properties as the wave numbers (perturbation of the Cs and Ca)

\[ j=1, 2 \text{ (fluid phases), } m=1, 2, 3 \text{ (wave numbers) } \]
Absence of A0-like mode ??

Partial branch only for S0

$V_{1\text{max}} = 3270 \text{ m/s}$
$V_{2\text{max}} = 550 \text{ m/s}$
$V_{3\text{max}} = 35 \text{ m/s}$
$V_{t\text{max}} = 1552 \text{ m/s}$

Complex « usual » modes obtained with real angles

Complex « unusual » modes obtained with imaginary angles

Measurable experimentally ??

K. E. Graff
Wave Motion in Elastic Solids 1975
Dispersion curves for Berea sandstone

- Obtained with the transition operator in relation with symmetrical modes

\[ T_S = (R + T - 1) / 2i \]

Enhancement of the vertical mode visualisation (related to the second dilatational wave)

Cut-off frequencies for symmetrical modes

\[ f_{jk} = (k + \frac{1}{2}) c_j / d \quad f_{ik} = k c_i / d \]

- \( k = \) positive integer
- \( j = 1, 2, 3 \)

\[ f_{10} = 16351, \quad f_{11} = 49053, \quad f_{12} = 81756, ... \]

\[ f_{10} = 15520, \quad f_{11} = 31040, \quad f_{12} = 46560, \quad f_{13} = 62080, ... \]
Angular resonances at \( f = 30 \text{ kHz} \)

Absence of spikes corresponding to the shear wave because of their small width of resonance
Effect of the total porosity variation on the transmission coefficient

- The spikes shift toward the low frequency
- The resonance width is more larger for a small porosity
Effect of permeabilities variation on the transmission coefficient

- The spikes shift toward the low frequency
- The increase of the transmission amplitude (above the second critical angle)

For prospects: study of parameters affecting the propagation of Lamb modes
Evaluation of the clogging by ultrasonics testing

- **Model for handling a layer of granular medium (Robu)**

![Diagram of a double porosity layer with fluid and aluminum plate](image)

- **Fabry-Perot method**:

\[
R_{g2} = R_{g1} + \frac{(T_{g1})^2 R_p}{1 - R_{g1} R_p}
\]

\[
T_{g2} = \frac{T_{g1} T_p}{1 - R_{g1} R_{dpp}}
\]

- Characterization of the clogging by injecting fines particles of clay with a water flow

Fluid – Aluminium plate – fluid

Fluid – DPP – fluid (previous study)

Fluid – Aluminium plate – fluid

\( R_{g1}, T_{g1} \)

\( R_{p}, T_{p} \)

D = 0 → The model presented
\[ C_{\text{exp}} = \frac{S}{S_0} \]

FFT of the temporal signal transmitted through the model filled with DPP

\[ C_{\text{num}} = \frac{T}{T_0} \]

FFT of the temporal signal transmitted through the model filled with water

Sensitivity of the acoustic measures to the clogging phenomenon
Conclusions

- Particularities observed for the fundamental modes in the dispersion curves of a dpp layer compared to an elastic layer.

- The third dilatational wave plays no role for the characterisation of the Lamb waves in this frequency range.

- This analytical study can be also of interest as a first approach for understanding the acoustic behaviour of cancellous bones (media with multiple porosities).

- The study of clogging by ultrasonic methods can lead subsequently to the development of non destructive methods of predicting phenomena affecting hydraulic structures.
Thank you for your attention