Microstructures for lowering the quarter wavelength resonance frequency of a hard-backed rigid-porous layer

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- For useful low frequency resonant absorption, what can be achieved using simple microstructures?
### Identical parallel uniform pores

**single pore**

Complex density: \( \rho(\omega) = \rho_0 / H(\lambda) \)

Complex compressibility: \( C(\omega) = (\gamma \rho_0)^{-1} [\gamma - (\gamma - 1) H(\lambda \sqrt{N_{PR}})] \)

<table>
<thead>
<tr>
<th>Pore Shape</th>
<th>( \lambda )</th>
<th>( H(\lambda) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>slit (width 2b)</td>
<td>( b \sqrt{(\omega / \nu)} )</td>
<td>( 1 - \tanh(\lambda \sqrt{-i}) / (\lambda \sqrt{-i}) )</td>
</tr>
<tr>
<td>cylinder (radius a)</td>
<td>( a \sqrt{(\omega / \nu)} )</td>
<td>( 1 - (2 / \lambda \sqrt{i}) J_1(\lambda \sqrt{i}) / J_0(\lambda \sqrt{i}) )</td>
</tr>
<tr>
<td>equilateral triangle (side d)</td>
<td>( (d \sqrt{3/4}) \sqrt{(\omega / \nu)} )</td>
<td>( 1 - 3 \coth(\lambda \sqrt{-i}) / (\lambda \sqrt{-i}) + 3i / \lambda^2 )</td>
</tr>
<tr>
<td>rectangle (sides 2a,2b)</td>
<td>( \frac{2ab}{\pi \sqrt{a^2 + b^2}} \sqrt{(\omega / \nu)} )</td>
<td>( \frac{-4i\omega}{\mu a^2 b^2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \left{ \alpha_k^2 \beta_l^2 \left( \alpha_k^2 + \beta_l^2 - \left( \frac{i \omega}{\mu} \right)^{-1} \right) \right}, ) ( \alpha_k = (k + \frac{1}{2}) \left( \frac{\pi}{a} \right), ) ( \beta_l = (l + \frac{1}{2}) \left( \frac{\pi}{b} \right) )</td>
</tr>
</tbody>
</table>

**bulk material**

\( \rho_b(\omega) = (T(\theta) / \Omega) \rho(\omega), \ C_b(\omega) = \Omega C(\omega) \)

\( T(\theta) = 1 / \cos^2(\theta) \)

\( K(\omega) = \omega \sqrt[\rho_b(\omega) C_b(\omega)]{\rho_b(\omega) C_{b}(\omega)} \)

\( Z(\omega) = (\rho_0 c_0^2)^{-1/2} \sqrt[\rho_b(\omega) C_b(\omega)]{\rho_b(\omega) / C_b(\omega)} \)

**hard-backed layer**

Surface impedance

\( Z(d) = Z_c \coth(-i kd) \)

\( \alpha(d) = 1 - \left| \left( R(d) \right)^2 \right| \)

Plane wave reflection coefficient

\( R(d) = \frac{\rho_0 c_0 - Z(d)}{\rho_0 c_0 + Z(d)} \)

**flow resistivity**

\( R_s = \frac{2 \mu T s_0}{\Omega r_h^2} \)

[Koseny – Carman]
Pore shape and inclination

0.03 m hard-backed thick layer, porosity 0.3

cylindrical pores of diameter $(2a) = 2$ mm

$R_{\text{cyl}} = \frac{8 \mu T}{\Omega a^2}$

$\theta = 0^\circ$

Fr$\text{e}$quency Hz

0 0.2 0.4 0.6 0.8 1

Normal incidence absorption coefficient

100 1000 10000

slit pores of width $(2b) = 2$ mm

$R_{\text{slit}} = \frac{3 \mu T}{\Omega b^2}$

Fr$\text{e}$quency Hz

0 0.2 0.4 0.6 0.8 1

Normal incidence absorption coefficient

100 1000 10000

rectangular pores with shortest dimension = 2 mm

$R_{\text{square}} = \frac{7.2 \mu T}{\Omega b^2}$

Fr$\text{e}$quency Hz

0 0.2 0.4 0.6 0.8 1

Normal incidence absorption coefficient

100 1000 10000

square

aspect ratio = 2

aspect ratio = 4

04/12/2017

SAPEM 2017
0.0264 m thick perforated metallic foam
from Chevillotte et al. JASA 2010
estimated JCA parameters: porosity 0.283, flow resistivity 8.4 kPa s m$^{-2}$, tortuosity 5.54, viscous characteristic length 0.281 mm, thermal characteristic length 1.54 mm

0.025 m thick simple microstructure - inclined slits
porosity 0.283, slit width 0.46 mm, edge-to-edge spacing 1.27 mm, angle $\theta = 65^\circ$
$\Rightarrow$ flow resistivity 17.28 kPa s m$^{-2}$, tortuosity 4.69
0.03 m thick layer with
dead end pore microstructure
from Leclaire et al. JASA 2015

- Surface porosity 0.0169
- Main pore radius 1.5 mm
- Dead end pore radius 1 mm
- Dead end pore length 20 mm
- N = 4 dead ends per period
- Thickness 0.03 m

Simple microstructure -
inclined cylindrical pores

- Bulk porosity 0.1
- Identical pores radius 1 mm
- Thickness 0.03 m

Graphs showing absorption coefficient vs. frequency for main pores only, dead end pores LF approx., and exact calculations.
layer thickness 25 mm
annular pores - outer radius 1 mm inner radius 0.75 mm porosity 0.1144; cylindrical pores radius 1 mm, porosity 0.2616

annular pores - outer radius 2 mm inner radius 1.25 mm porosity 0.1623; cylindrical pores radius 2 mm, porosity 0.2663

Nori and Venegas JASA 141 (2017)
log-normal distributions of slit widths

median slit width 2 mm
standard deviation 0.5 \( \phi \) units

Yamamoto and Turgut, JASA 1998

Horoshenkov et al, JASA 2016

3-cm thick hard-backed rigid porous layer with porosity 0.3

Normal incidence absorption coefficient

0°

45°

uniform
non-uniform
identical
Slits with sinusoidal sides


Constraints:

- \( \frac{a}{2b} \leq 0.1 \)
- \( X > a \)
- \( a > 0.1 \text{ mm} \)

Flow resistivity \( \text{Pa s m}^{-2} \)

\[
\frac{a}{2a}, \frac{b}{2b} \approx 0.2 \text{ mm}, b = 1 \text{ mm}, \Omega = 0.3
\]
sinusoidally-sided and inclined slits

$X = 0.3 \text{ mm, } d = 30 \text{ mm, } a = 0.2 \text{ mm, } b = 1 \text{ mm, porosity 0.3}$
**dual-porosity slits**


- **0°**: meso-slits, 2 mm wide, edge-to-edge spacing 8 mm; micro-slits, 0.2 mm wide, edge-to-edge spacing 1.4 mm; total porosity 0.3; layer thickness 0.03 m.

- **70°**: meso- and micro-slits, identical slits.

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**Diagram Note**

- $2b_{me}$: meso-slit
- $2b_{mi}$: micro-slit

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**Text Note**

- Mesio-slits, 2 mm wide, edge-to-edge spacing 8 mm; micro-slits, 0.2 mm wide, edge-to-edge spacing 1.4 mm; total porosity 0.3; layer thickness 0.03 m.
Summary of predicted quarter wavelength resonances of 0.03 m thick layers with porosity 0.3 for various microstructures

<table>
<thead>
<tr>
<th>Microstructure</th>
<th>Frequency kHz</th>
<th>amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identical slits width 2 mm @ 0°</td>
<td>2.95</td>
<td>0.27</td>
</tr>
<tr>
<td>Identical slits width 2 mm @ 70°</td>
<td>0.95</td>
<td>0.8</td>
</tr>
<tr>
<td>Identical cylinders diameter 2mm @ 0°</td>
<td>2.95</td>
<td>0.47</td>
</tr>
<tr>
<td>Identical cylinders diameter 2mm @ 70°</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Sinusoidal slits width 2 ± 0.4 mm @ 0°</td>
<td>2.8</td>
<td>0.28</td>
</tr>
<tr>
<td>Sinusoidal slits width 2 ± 0.4 mm @ 70°</td>
<td>0.8</td>
<td>0.95</td>
</tr>
<tr>
<td>Dual porosity slits widths 2 and 0.2 mm @ 0°</td>
<td>2.7</td>
<td>0.75</td>
</tr>
<tr>
<td>Dual porosity, sinusoidal meso-slits @ 0°</td>
<td>2.3</td>
<td>0.85</td>
</tr>
<tr>
<td>Dual porosity - slit widths 2 and 0.2 mm @ 70°</td>
<td>0.85</td>
<td>0.9</td>
</tr>
</tbody>
</table>
CONCLUSIONS

• The frequency of the quarter wavelength resonance of a hard-backed layer with low values of porosity and flow resistivity is decreased by increasing tortuosity.

• Tortuosity is increased by inclining pores with respect to the surface normal, by varying pore widths in a sinusoidal manner, by a dual porosity arrangements and by combinations of these.

• Identical inclined pores give comparable absorption spectra to those due to comparatively complicated microstructures.

• If the pores are annular and cylindrical or have log-normal width distributions, inclining the pores lowers the quarter wavelength resonance by less than other methods.

• Useful low frequency resonant absorption can be achieved by materials with simple microstructures suitable for manufacture by 3D printing.
cylindrical annular pores
Nori and Venegas JASA 141 (2017)

Johnson-Champoux-Allard-Lafarge formulation

viscous and thermal characteristic lengths and permeabilities are equal

\[ \rho_b(\omega) = T \rho_0 \left[ 1 + \frac{i R_A^2 \Omega}{\omega \rho_0 T} G(R, r, L, T) \right] \]

\[ C_b(\omega) = (\gamma P_0)^{-1} \left[ \gamma - (\gamma - 1) \left[ 1 + \frac{i R_A^2 \Omega}{\omega \rho_0 T N_{PR}} G'(R, r, L, T) \right] \right]^{-1} \]

\[ G(R, r, L, T) = \sqrt{1 - \frac{4i T \eta \rho_0 \omega}{R_A^2 R^2 (1 - r)^2 \Omega^2}} \quad G'(R, r, L, T) = \sqrt{1 - \frac{4i T \eta \rho_0 \omega N_{PR}}{R_A^2 R^2 (1 - r)^2 \Omega^2}} \]

\[ \Omega = \pi R^2 (1 - r^2)/L^2 \]

\[ Y(r) = 1 + r^2 - (1 - r^2)/\ln(1/r) \]

\[ R_A = \frac{8T \eta}{\Omega R^2 Y(r)} \]
Dual porosity: meso-pores with sinusoidally-varying sides

- **Parallel to surface normal**
  - Dual porosity with sinusoidally-sided meso-slits and smooth micro-slits

- **45° to surface normal**
  - Dual porosity with sinusoidally-sided meso-slits

- **70° to surface normal**
  - Dual porosity with sinusoidally-sided meso-slits and smooth micro-slits

Absorption coefficient vs. Frequency (Hz)
Melamine foam

Chevillotte et al JASA 128 1775 (2010)

- 50 mm thick, porosity 0.99, normal-to-surface slits with log-normal distribution of widths, median width $1.5 \times 10^{-4}$ m, SD 1 φ-unit

JCAL:
- 56.7 mm thick
- porosity 0.99
- flow resistivity 12 kPa s m$^{-2}$
- tortuosity 1.01
- viscous characteristic length 100 μm
- thermal characteristic length 400 μm
- thermal permeability $1.5 \times 10^{-9}$ m$^2$
Slits with regular discrete width variation

\[ T = \frac{(l_1 S_1 + l_2 S_2)(l_2 S_1 + l_1 S_2)}{(l_1 + l_2)^2 S_1 S_2} \]

Assumes constant velocities in each section

\[ S_1 = 2(a + b), \quad S_2 = 2b \]

Equation (5.A.7) p.104 in Allard and Atalla text

<table>
<thead>
<tr>
<th>(a/b (b = 0.001 \text{ m}))</th>
<th>(l_2/l_1)</th>
<th>tortuosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.1</td>
<td>1.003</td>
</tr>
<tr>
<td>2</td>
<td>0.9</td>
<td>1.66</td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
<td>2.59</td>
</tr>
<tr>
<td>8</td>
<td>0.9</td>
<td>4.54</td>
</tr>
</tbody>
</table>