On the use of uncertainties from the characterization to the computation of dispersion envelope of poro-elastic media

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Thanks to Stephen Hillenburg and Matthew Edwards

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Outline

Introduction

- The modelling of the acoustical behaviour of a poro-elastic material usually requires **11 parameters** (6 JCAL + 4 Elastic + thickness)
- **Each parameter** is measured or characterized with a given **uncertainty**
- Several samples per material $\longrightarrow$ **standard deviations** for each parameter
- The idea of this work is to propose a **methodology** to take into account both measurement uncertainties and standard deviations between samples **to simulate envelopes** in place of a single average curve.
- 2 methods are investigated: the **complete method** and the **sampling method**.

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Overview of the used characterisation method

Direct methods

- Air flow resistivity measurement
  - ISO 9053
- Porosity measurement
  - Beranek, Salissou & Panneton

Indirect methods: (Audible range) Analytical inversion

- for porous materials : Olny & Panneton, from measurement of ρ_{eq} and K_{eq}
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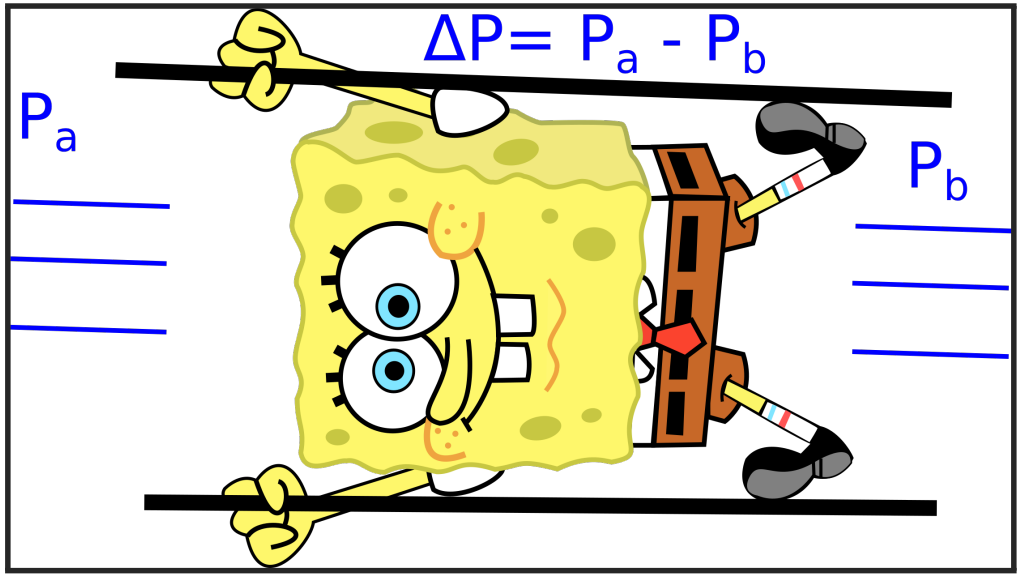
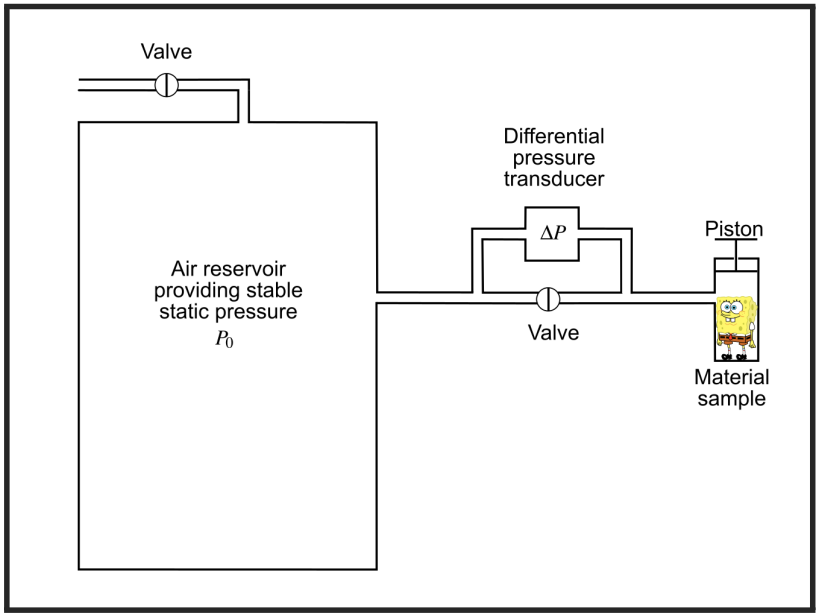
Static air flow resistivity

- ISO 9053

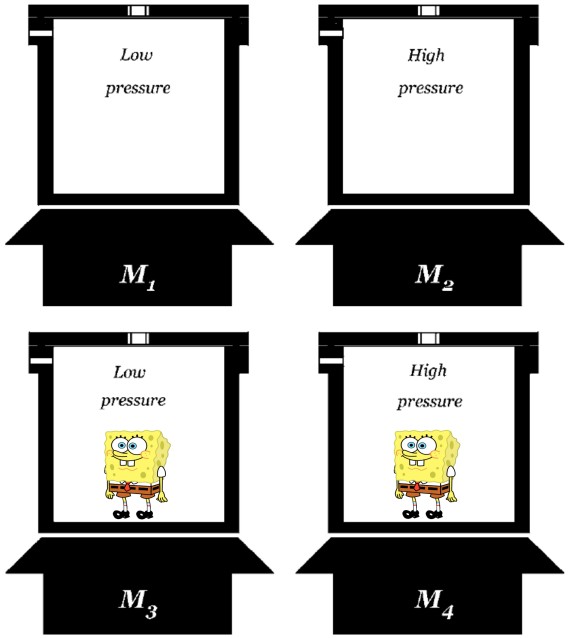


Open porosity

Beranek *et al.* 1942, further modified by Champoux, Stinson and Daigle 1991

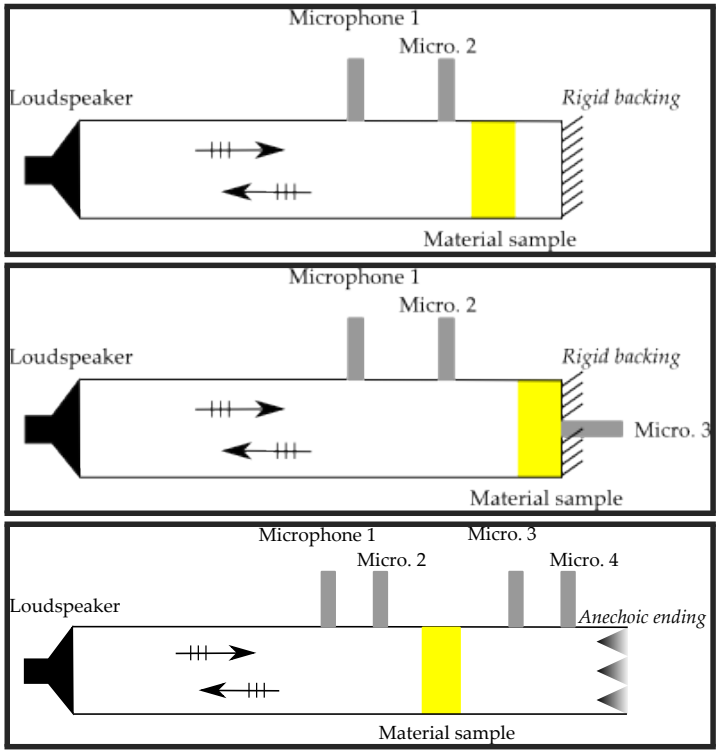


Salissou & Panneton



Impedance tube setups for measurement of ρ_{eq} et K_{eq}

- 2 load method : Utsuno *et al.*
- 3 mic. method : Iwase *et al.*
- 4 mic. method : Song & Bolton



Analytical inversion for porous materials : α_∞, Λ, Λ', k'_0

From measurement of ρ_{eq} and K_{eq}, separate assessment of the visco-inertial and thermal effects following the Johnson-Champoux-Allard-Lafarge model

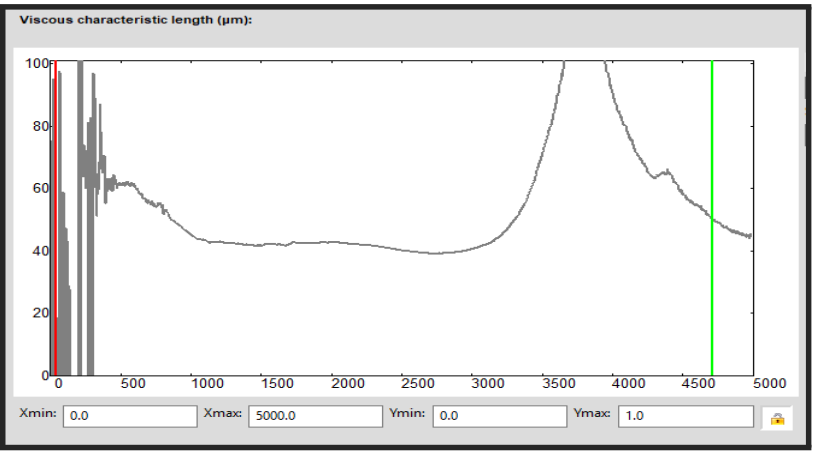
$$\widetilde{\rho_{eq}} = \frac{\alpha_\infty \rho_0}{\phi}\left[1 - \mathrm{j}\frac{\sigma\phi}{\omega\rho_0\alpha_\infty}\sqrt{1 + \mathrm{j}\frac{4{\alpha_\infty}^2\eta\rho_0\omega}{\sigma^2\Lambda^2\phi^2}}\right] \quad \text{or} \quad \widetilde{\rho_{eq}} = \widetilde{A} + \mathrm{j}\widetilde{B}$$

$$\widetilde{K_{eq}} = \frac{\gamma P_0/\phi}{\gamma - (\gamma-1)\left[1 - \mathrm{j}\frac{\phi\kappa}{k'_0 C_p\rho_0\omega}\sqrt{1 + \mathrm{j}\frac{4{k'_0}^2 C_p\rho_0\omega}{\kappa{\Lambda'}^2\phi^2}}\right]^{-1}} \quad \text{or} \quad \widetilde{K_{eq}} = \widetilde{C} + \mathrm{j}\widetilde{D}$$

$$\alpha_\infty = \frac{\phi}{\rho_0}\left(\mathrm{Re}(\widetilde{\rho_{eq}}) - \sqrt{\left[\mathrm{Im}(\widetilde{\rho_{eq}})\right]^2 - \frac{\sigma^2}{\omega^2}}\right) \quad \text{and} \quad \Lambda = \frac{\alpha_\infty}{\phi}\sqrt{\frac{2\eta\rho_0}{\omega\mathrm{Im}(\widetilde{\rho_{eq}})}\left[\alpha_\infty\rho_0/\phi - \mathrm{Re}(\widetilde{\rho_{eq}})\right]}$$

$$k'_0 = \frac{\phi\kappa}{C_p\rho_0\omega}\Big/\sqrt{-\mathrm{Re}\left(\left[\frac{\gamma P_0 - \phi\widetilde{K_{eq}}}{\gamma P_0 - \gamma\phi\widetilde{K_{eq}}}\right]^2\right)} \quad \text{and} \quad \Lambda' = 2\sqrt{\frac{\kappa}{C_p\rho_0\omega}\left\{-\mathrm{Im}\left(\left[\frac{\gamma P_0 - \phi\widetilde{K_{eq}}}{\gamma P_0 - \gamma\phi\widetilde{K_{eq}}}\right]^2\right)\right\}^{-1}}$$

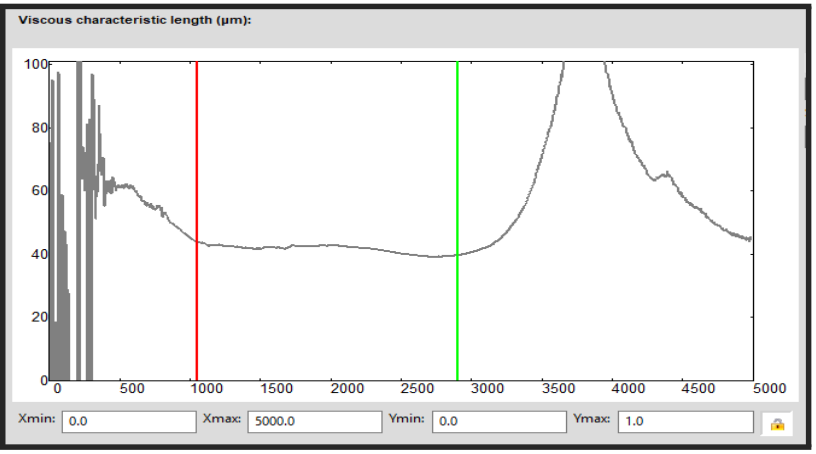
Static error estimation from parameters' spectra



	Value	*std*
*Mode**	42	
Mean	56	22

*Mode: the value in a given dataset that appears most frequenly

Viscous characteristic length (μm) : standard deviation estimation from a **large frequency interval**



	Value	*std*
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Viscous characteristic length (μm) : standard deviation estimation from a **small frequency interval**

Measurement uncertainties and standard deviations between samples

- **Common method** :
  $\forall i \in [1, n]$, i sample index, and N_i the frequency number,
  $std_m = \sqrt{\frac{\sum_1^n (\bar{x_i} - \bar{X_m})^2}{n-1}}$ with $\bar{X_m} = \frac{1}{n} \sum_1^n \bar{x_i}$
  and on each sample $std_i = \sqrt{\frac{\sum_1^{N_i} (x_i - \bar{x_i})^2}{N_i - 1}}$
- **Exact expression** :
  $std = \sqrt{\frac{\sum_1^n \sum_1^{N_i} (x_i - \bar{X})^2}{\sum_1^n N_i - 1}}$ with $\bar{X} = \frac{1}{\sum_1^i N_i} \sum_1^n \sum_1^{N_i} x_i$
- **Proposed method** :
  Hypothesis: $N_1 = N_2 = ... = N_n = N$ and N big enough
  $std \approx \sqrt{\frac{\sum_1^n std_i^2}{n} + std_m^2}$

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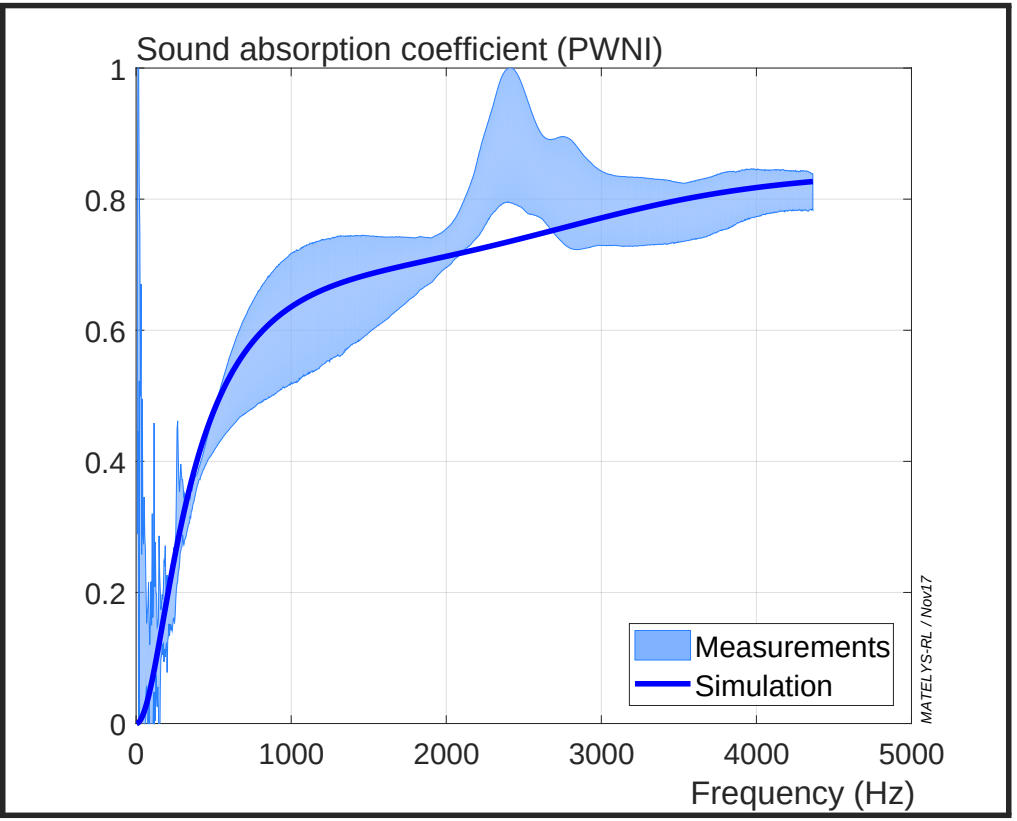
Measurement uncertainties and standard deviations between samples

	Common method	*Proposed method*
$\begin{cases} \Lambda_{sample1} = 142 \pm 40\mu m \\ \Lambda_{sample2} = 142 \pm 60\mu m \end{cases}$	$142 \pm 0\mu m$	$142 \pm 51\mu m$

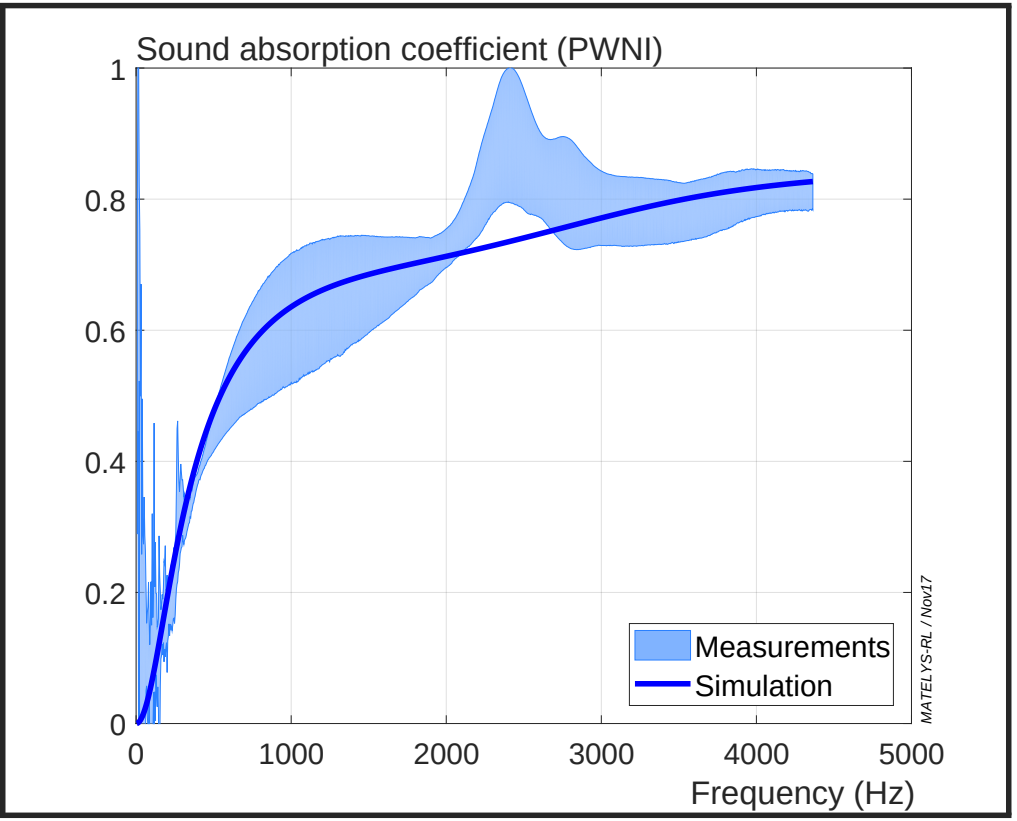
Comparison between the common and the proposed method :
Example for the viscous characteristic lengths of 2 samples

Single average curve versus enveloppe



- Foam modeled with **JCAL parameters** given by:
  thickness $= 30.00 \pm 0.22$ mm,
  $\phi = 0.99 \pm 0.01$,
  $\sigma = 109\ 200 \pm 41\ 000$ N.s.m^{-4},
  $\Lambda = 13 \pm 5\ \mu$m,
  $\Lambda' = 105 \pm 19\ \mu$m,
  $\alpha_\infty = 1.13 \pm 0.13$
- A simulated envelope would be more appropriate than the simulation with mean values.
- Other indicators as absorption or transmission under a diffused field could be investigated.

Single average curve versus enveloppe



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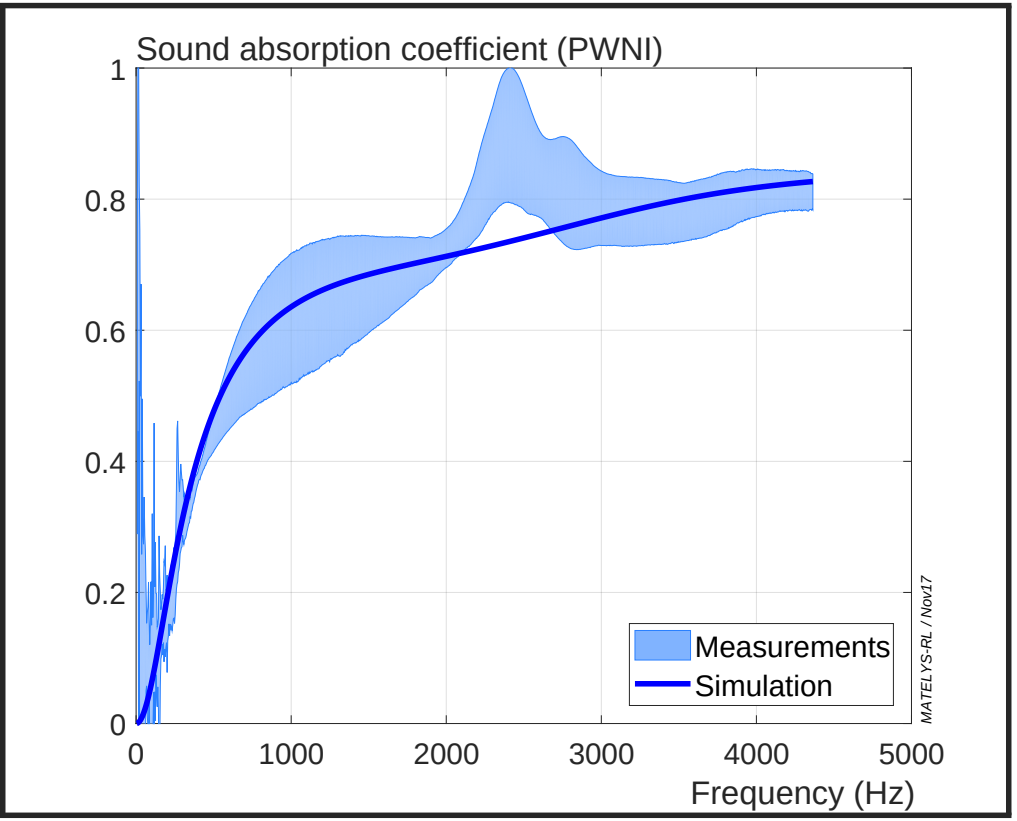

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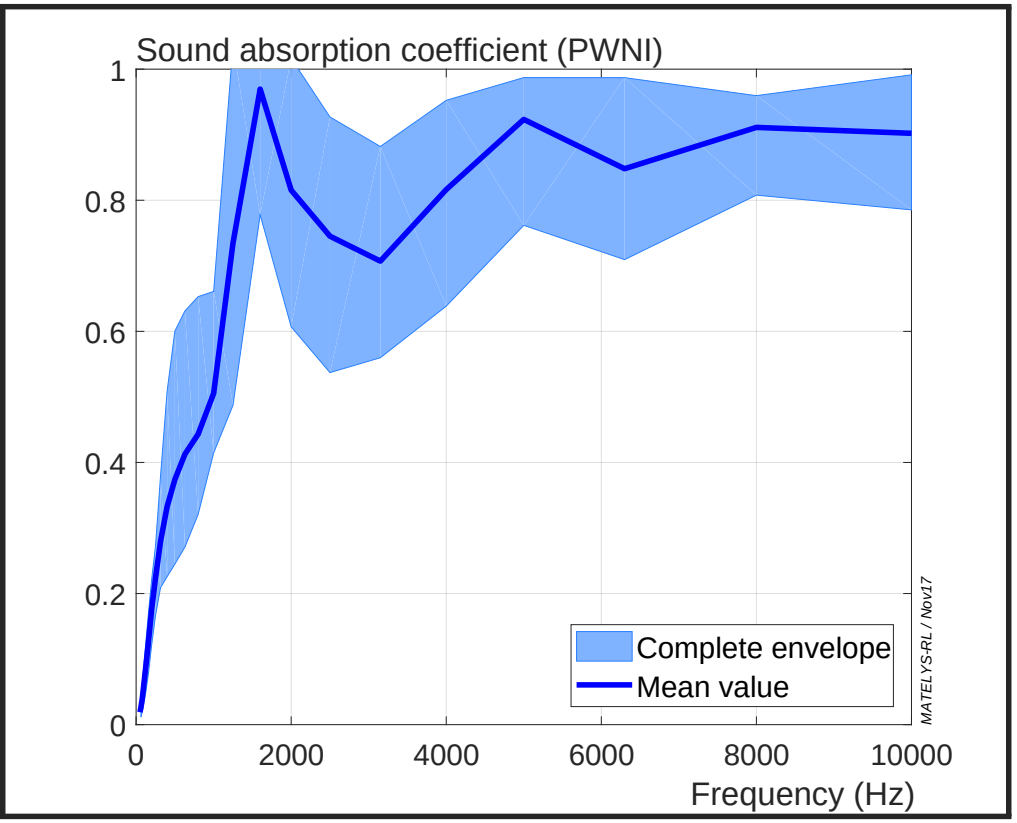

- **N parameters** - Example for JCAL/elastic model ⇒ 11 parameters
- 2 possibilities per parameter, **minimum and maximum values** - Example for $\Lambda = 13 \pm 5\ \mu\mathrm{m} \Rightarrow \Lambda_{min} = 8\ \mu\mathrm{m}$ and $\Lambda_{max} = 18\ \mu\mathrm{m}$
- **All combinations** are computed - Example for JCAL/elastic model ⇒ $2^{11} = 2048$ cases to compute
- All these simulations lead to **a simulated envelope** but can be costly

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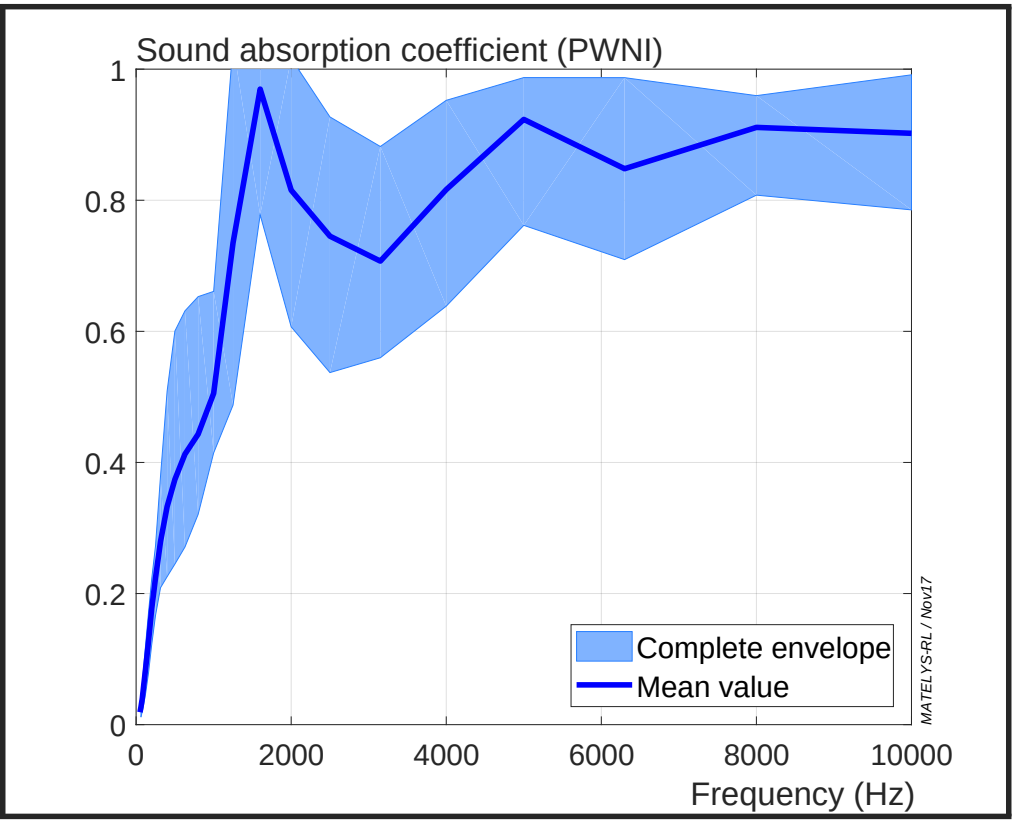

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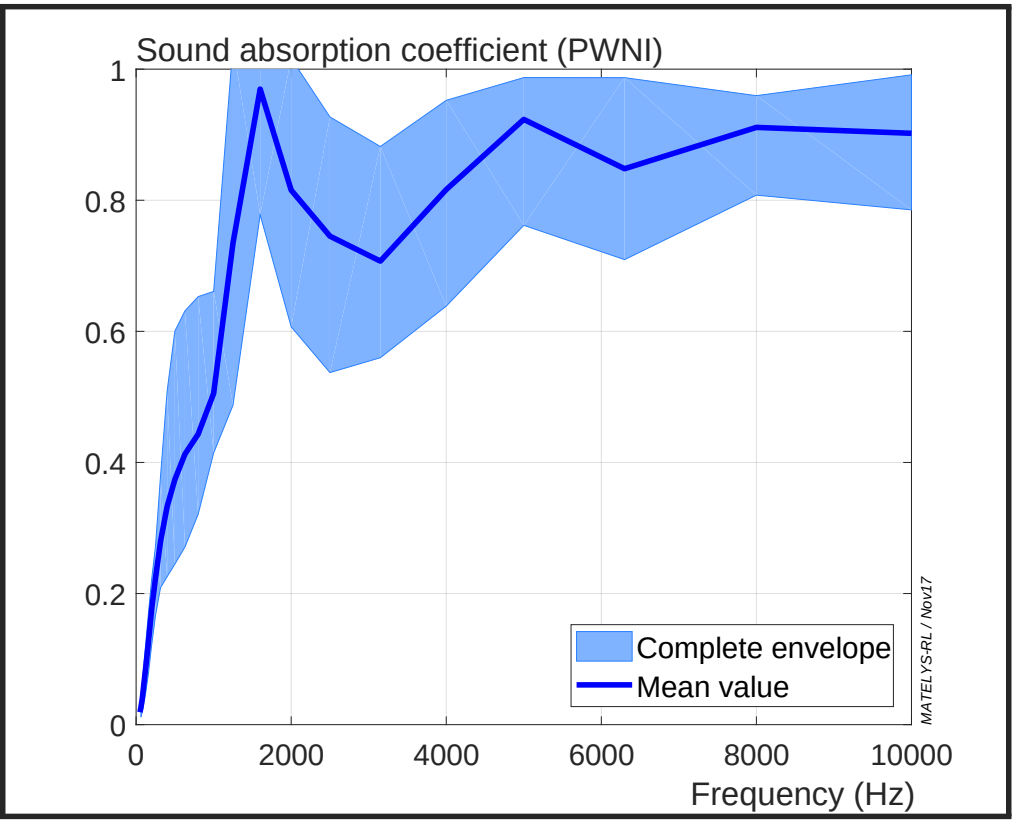

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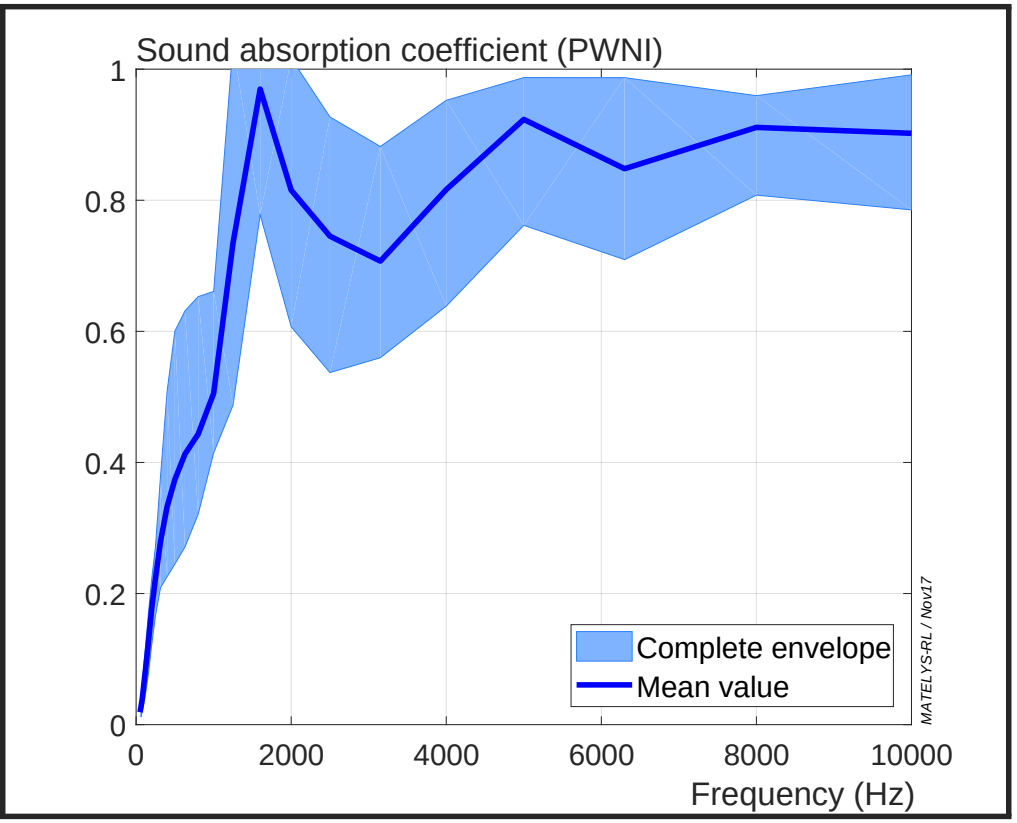

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Computation cost

3 types of quantities:

- the sound absorption coefficient under a diffuse field excitation α_{DF},
- the transmission loss under a diffuse field excitation TL_{DF}
- the surface impedance under a plane wave at normal incidence Z_{NI}.

Configuration	*Nb of simul.*	α_{DF}	$\mathbf{T}L_{DF}$	$\mathbf{Z}_{NI}$
1 sample	1	3.15 s	14.49 s	1.23 s
1 envelope	2 048	1.78 h	8.25 h	0.70 h
800 envelopes	1 638 400	1 434 h	6 595 h	560 h

Complete computation cost: Computational time may be prohibitive for systematic envelope evaluation for a large number of samples.

Computer capacities: processor Intel(R) Core(TM) i5-4200M CPU @ 2.50GHz and 8 GB of RAM

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- For each sample, parameters are **randomly chosen in a range** defined by:
  $\forall P$ parameter $\mid P \in [\bar{X}_{param} - \beta \times std_{param}\ ,\ \bar{X}_{param} + \beta \times std_{param}]$
  with β a coefficient $\beta \in [1, 3]$
- **Dismiss unrealistic samples** - Example: a sample cannot have a negative resistivity even if its resistivity equals 40 000 ± 50 000 $N.s.m^{-4}$.
- Experience showed that **20 samples** are enough to simulate one envelope,
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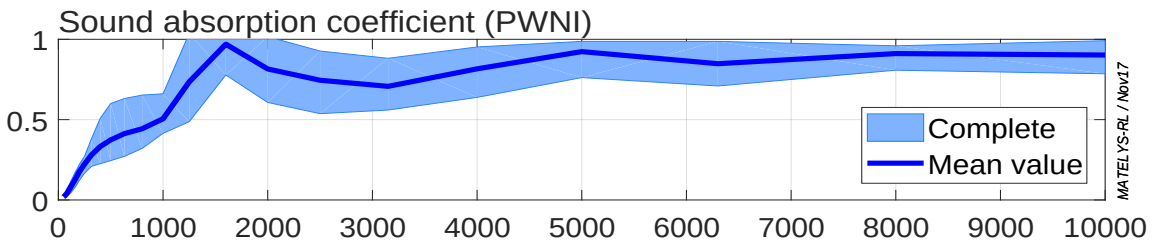
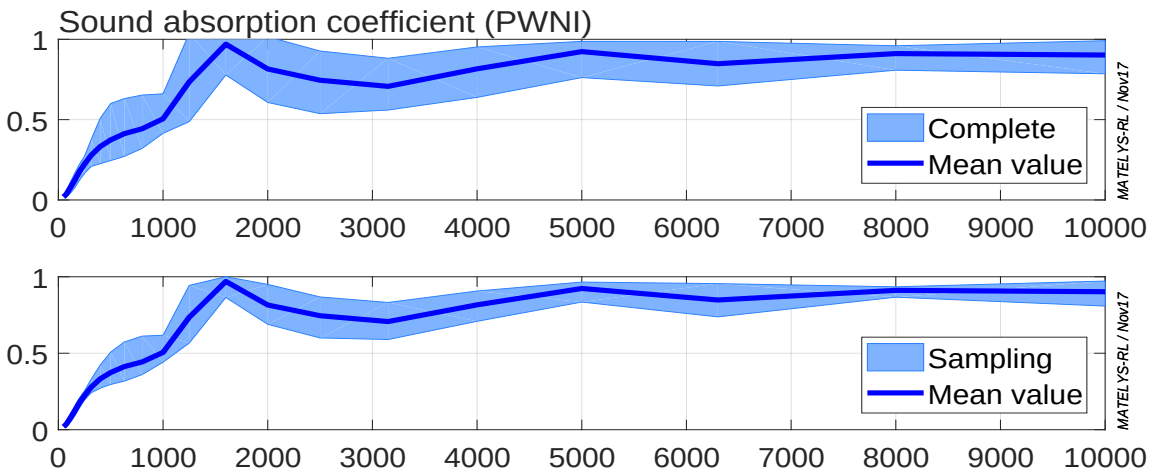
β influence



$$P = \bar{X}_{param} \pm \beta \times std_{param}$$

β influence



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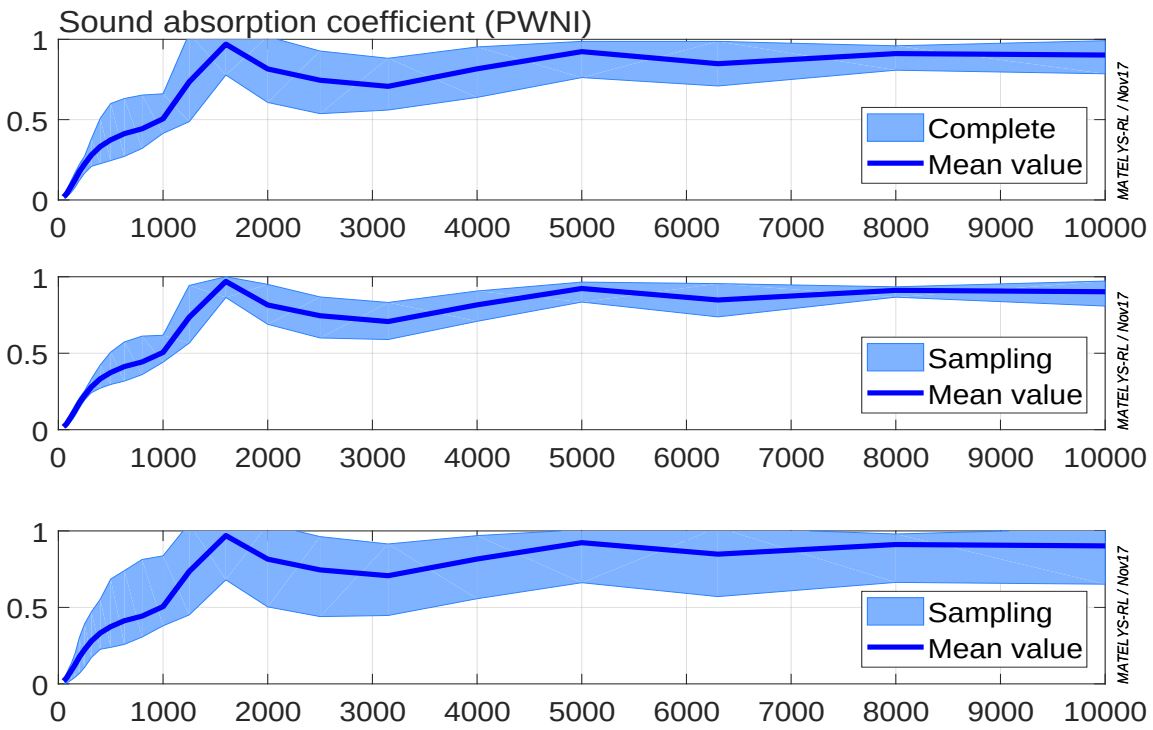
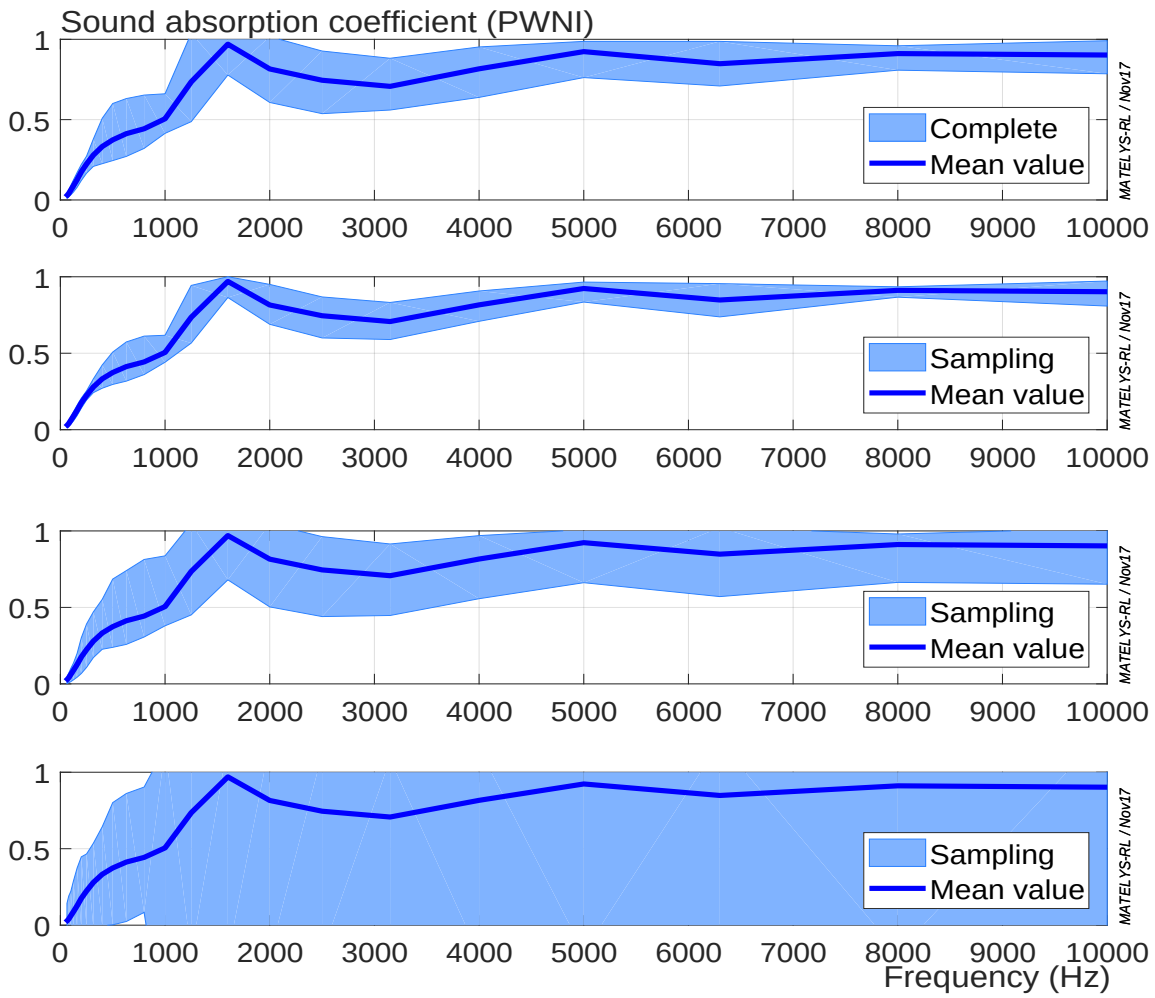


$$P = \bar{X}_{param} \pm \beta \times std_{param}$$

$$\beta = 1$$

$$\beta = 2$$

β influence



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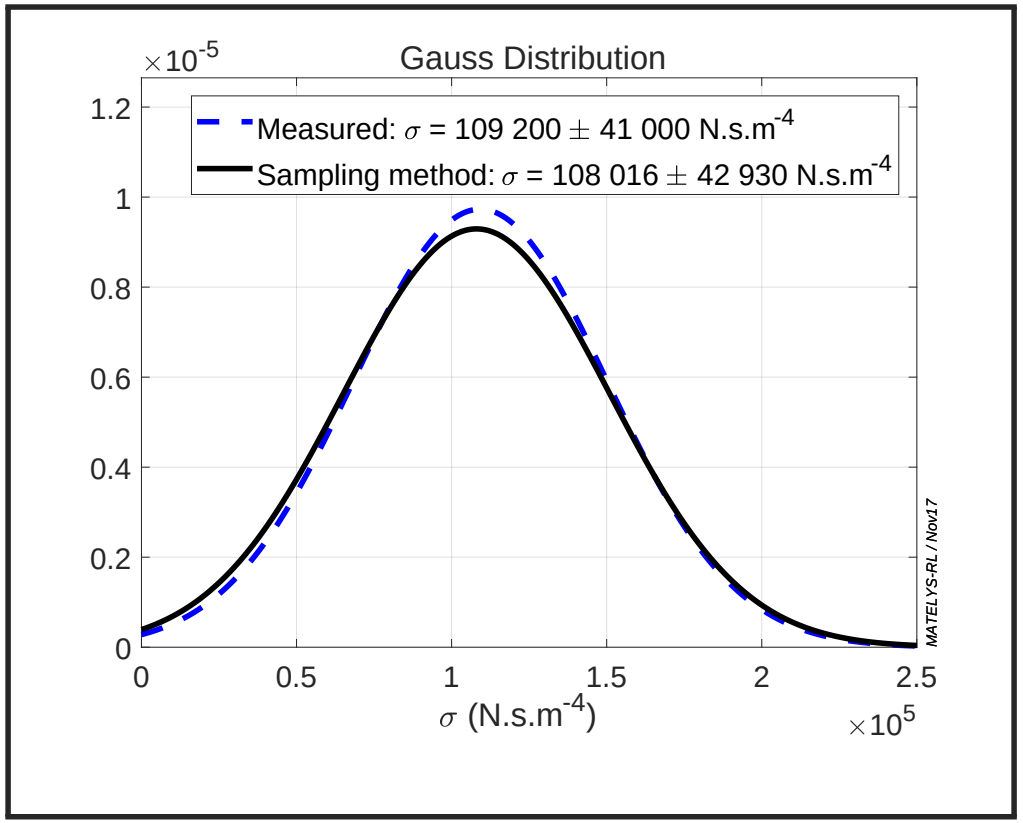
$$\beta = 1$$

$$\beta = 2$$

$$\beta = 3$$

Automatic sampling method



Automatic sampling method principle:

- An **iterative procedure** is used to find the best β,
- the algorithm is stopped when the **difference** between the generated population and the measured one is **less than 5% for both the mean value and the standard deviation**.
- the selection of samples is done before running computations so its **time cost is negligible**.
- **Tested** on a database filled with **800 materials**.
- The sampling method generates envelopes **similar** to the complete method.

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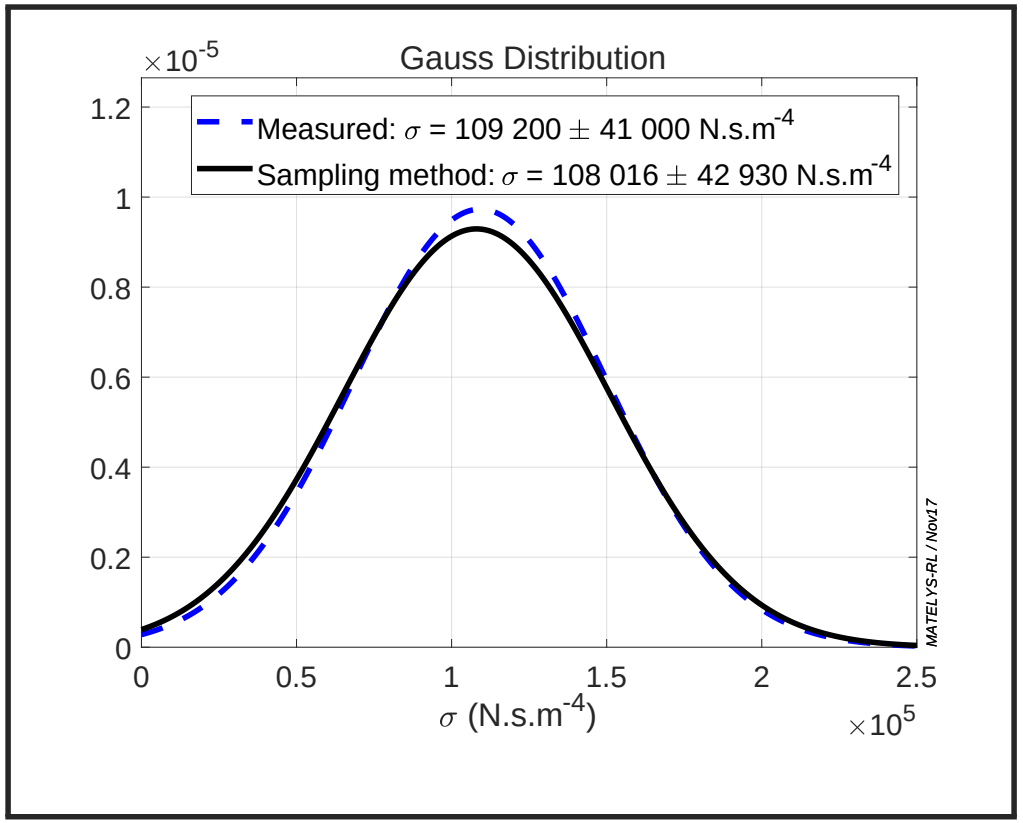

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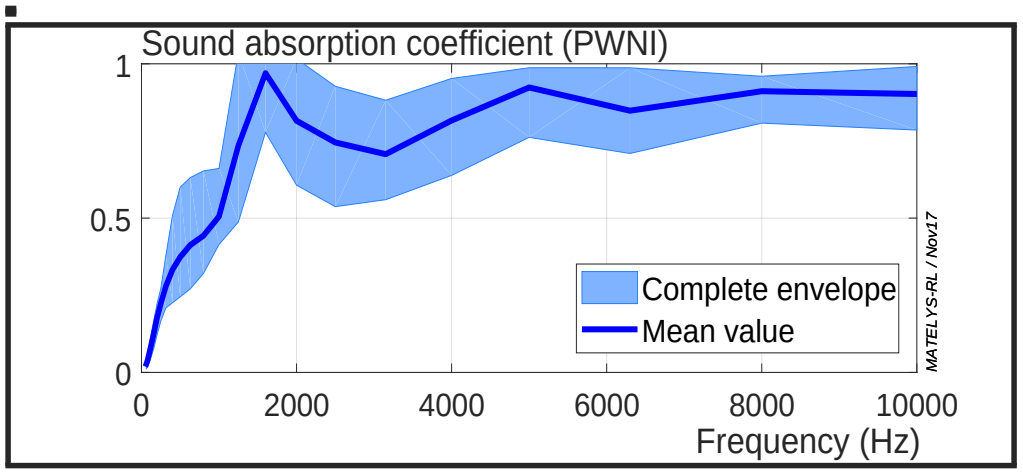

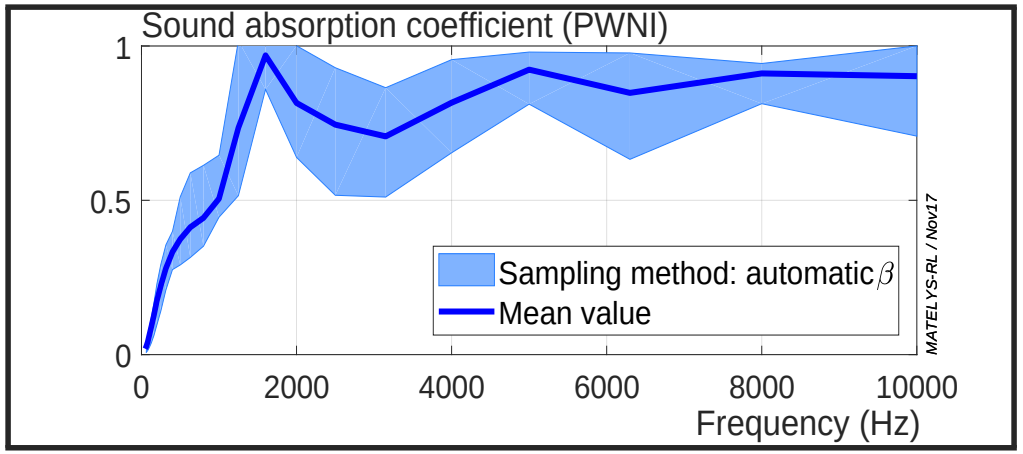

Automatic sampling method principle:

- An **iterative procedure** is used to find the best β,
- the algorithm is stopped when the **difference** between the generated population and the measured one is **less than 5% for both the mean value and the standard deviation**.
- the selection of samples is done before running computations so its **time cost is negligible**.
- **Tested** on a database filled with **800 materials**.
- The sampling method generates envelopes **similar** to the complete method.

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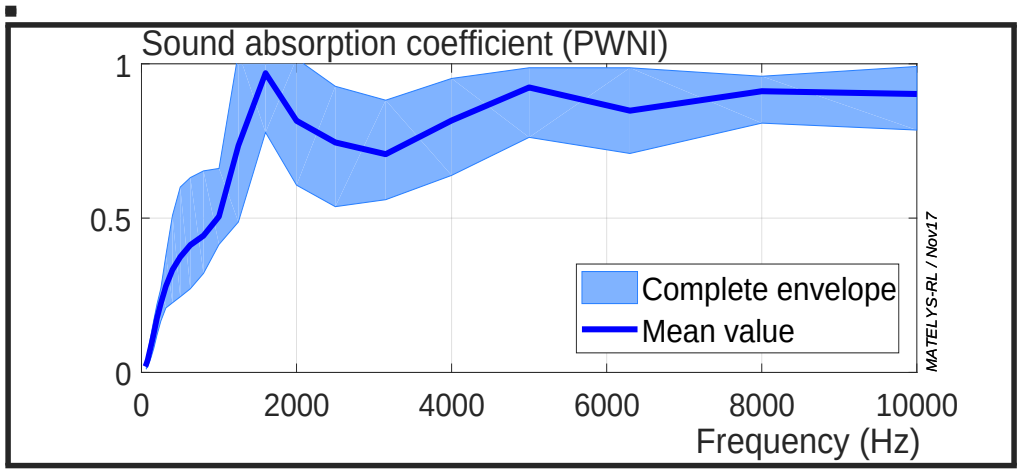

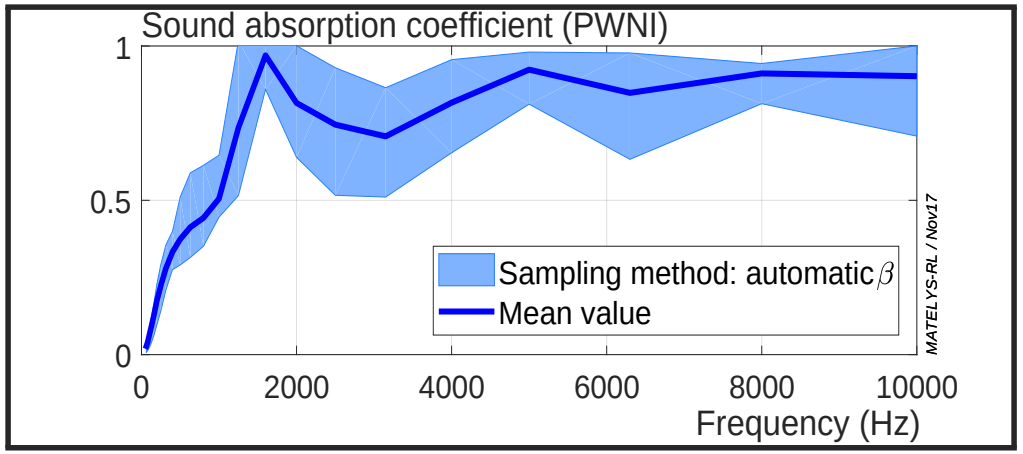

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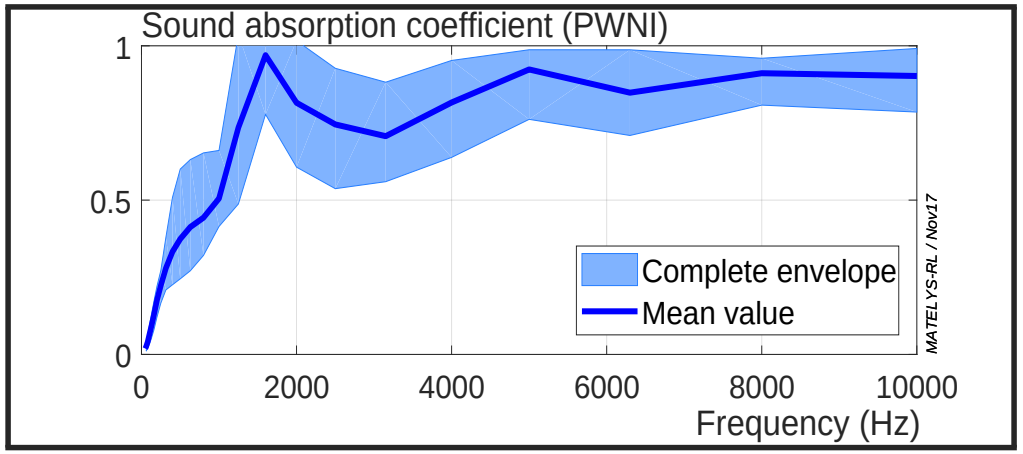

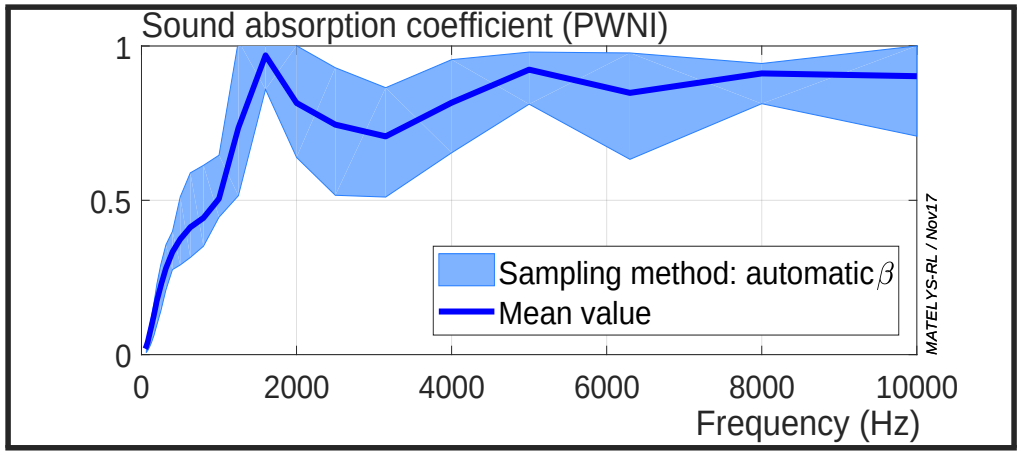

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Sampling method calculation cost

Configuration	*Nb of simul.*	α_{DF} +	$\mathbf{T}L_{DF}$ +	$\mathbf{Z}_{NI}$	= Σ
1 sample	1	3.15 s	14.49 s	1.23 s	18.87 s
Sampling method 1 envelope	20	1.05 min	4.83 min	0.41 min	6.28 min
Sampling method 800 envelopes	16 000	14.00 h	64.40 h	5.47 h	83.87 h
Complete method 800 envelopes	1 638 400	1 434 h	6 595 h	560 h	8 589 h

The sampling method enables to **divide the calculation cost** of the complete computation **by a factor 100**.

Conclusion

- A methology has been proposed to take into account for both measurement uncertainties and standard deviation between samples in order to efficiently **simulate envelopes in place of a single average curve**.
- **Similar envelopes** are obtained using two different methods: a complete method which uses minimum and maximum values for each parameter, and a sampling method using only few representative samples
- The sampling method enables to **divide the calculation cost** of the complete computation **by a factor 100**.
- This method could be used with other types of simulation.

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