Acoustic Response of Anisotropic Multilayered Structures

Sub-layering of the anisotropic poroelastic core and influence of the material natural axis orientation

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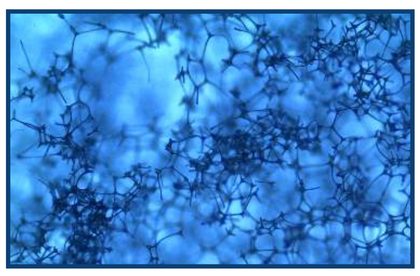
Motivation

- Open-cell porous materials are inherently anisotropic due to manufacturing processes, comprising a highly irregular microscopic structure.
- The intrinsic anisotropy of poroelastic media presents a potential for the optimal design of multilayered systems for multifunctional performances.
- There is a need for modelling tools to study the influence of the inherent mechanical and acoustic anisotropy of poroelastic materials on the acoustic behaviour of anisotropic multilayered structures.



Microstructure of a melamine foam (x20).

Objectives

- A plane wave method is presented to study the influence of material orientation on the dynamic behaviour of multilayered panels including anisotropic poroelastic media.
- A state vector approach is used and the solution is obtained recursively as a a superposition of waves in each layer[1]. This allows to obtain the acoustic and mechanical solutions of anisotropic multilayered systems under single-wave excitation with arbitrary angle of incidence.
- Performance indicators of the multilayered system and its composing layers derive therefrom, such as powers dissipated by each individual material layer.
- The model has been extended to include the computation of the acoustic response of multilayer systems under an acoustic diffuse field excitation.

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Problem: Dynamic behaviour of an homogeneous layer

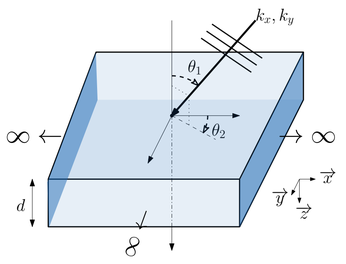


Fig. 1 - Layer of thickness d and infinite surface excited by a plane wave at an arbitrary angle of incidence

The behaviour of a material layer may be described using dynamic equations and constitutive laws.
For an anisotropic poroelastic media these are:

$$\nabla\cdot\hat{\sigma}^s = -\omega^2\tilde{\rho}_s\mathbf{u}^s - \omega^2\tilde{\rho}_{eq}\tilde{\gamma}\mathbf{u}^t$$
$$-\nabla p = -\omega^2\tilde{\rho}_{eq}\tilde{\gamma}\mathbf{u}^s - \omega^2\tilde{\rho}_{eq}\mathbf{u}^t$$
$$\hat{\sigma}^s = \mathbf{H}\,\varepsilon^s$$
$$p = -\tilde{K}_{eq}\nabla\cdot\mathbf{u}^t$$

which can be simplified as

$$\left(\mathbf{A}_0 + \mathbf{A}_x\frac{\partial}{\partial x} + \mathbf{A}_y\frac{\partial}{\partial y} + \mathbf{A}_z\frac{\partial}{\partial z}\right)\mathbf{w}(x,y,z) = \mathbf{0}$$

vector of all variables

Solution: Plane wave approach

Every variable may be expressed in the form:

$$v(x,y,z) = v(z)e^{\imath(\omega t - k_x x - k_y y)}$$

Thus, the behaviour of the layer can be written as

$$\left(\mathbf{R} + \mathbf{A}_z\frac{\partial}{\partial z}\right)\mathbf{w}(z) = \mathbf{0}$$

To solve the system of equations we partition vector of all variables into:

$$\mathbf{w}(z) = \begin{Bmatrix}\mathbf{s}(z)\\ \mathbf{s}'(z)\end{Bmatrix} = \begin{Bmatrix}\text{state vector}\\ \text{remaining variables}\end{Bmatrix}$$

thus to solve the problem is to solve for the state vector through the state-space representation:

$$\frac{\partial}{\partial z}\mathbf{s}(z) = -\boldsymbol{\alpha}\,\mathbf{s}(z)$$

STATE MATRIX

The response at any point z may be computed from the transfer matrix and the state vector at a know point z_0

$$\mathbf{s}(z) = \mathbf{e}^{-(z-z_0)\boldsymbol{\alpha}}\mathbf{s}(z_0)$$

How to compute the state matrix $\boldsymbol{\alpha}$?

Method: State Matrix

The generalised un-symmetric eigenvalue problem associated to $\mathbf{R}$ and $\mathbf{A}_z$ can be written as

$$\mathbf{A}_z = \mathbf{R}\Phi\Lambda\Psi, \qquad \Psi = \Phi^{-1}$$

Due to the rank deficiency of the problem, the eigenvectors and eigenvalues may be separated into two sets:

$$\Lambda = \begin{bmatrix}\Lambda_e & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}, \qquad \Phi = [\Phi_e \;\; \Phi_0], \qquad \Psi = \begin{bmatrix}\Psi_e\\ \Psi_0\end{bmatrix}$$

The system of equations describing the state of the layer may be then expressed as

$$\mathbf{R}_0\left(\mathbf{I}_m - \Phi_e\Lambda_e\left(\mathbf{R}_e\Phi_e\Lambda_e\right)^{-1}\mathbf{R}_e\right)\mathbf{w}(z) = \mathbf{0}$$

Where m is the length of $\mathbf{w}(z)$, and

$$\mathbf{R}_e = \Psi_e\mathbf{R}, \qquad \mathbf{R}_0 = \Psi_0\mathbf{R}$$

By partitioning the latter into $\begin{bmatrix}\mathbf{B}_0 & \mathbf{B}_s\end{bmatrix}\mathbf{w}(z) = \mathbf{0}$, a direct relation between the state vector and the remaining variables may be established as $\mathbf{s}_0(z) = -\mathbf{B}_0^{-1}\mathbf{B}_s\mathbf{s}(z)$.
The vector containing all variables is thus expressed exclusively as a function of the state vector $\mathbf{w}(z) = \mathbf{T}\mathbf{s}(z)$, with

$$\mathbf{T} = \begin{bmatrix}\mathbf{I}_n\\ -\mathbf{B}_0^{-1}\mathbf{B}_s\end{bmatrix}$$

The state matrix for a linear homogeneous generally anisotropic media as

$$\boldsymbol{\alpha} = \left(\Psi_e\mathbf{A}_z\mathbf{T}\right)^{-1}\Psi_e\mathbf{R}\mathbf{T}$$

Outcome: Acoustic and mechanical indicators

From the state-space representation, the state vector may be expressed as a function of the eigenvalues and eigenvectors

$$\mathbf{s}(z) = \boldsymbol{\Phi}\,\mathbf{e}^{-\boldsymbol{\Gamma}z}\,\mathbf{q}$$

which may be interpreted as a superposition of waves in the medium, where:

- $\boldsymbol{\Phi}$ are the wave polarisations,
- $\mathbf{e}^{-\Gamma z}$ describes the propagation of the waves ,
- $\mathbf{q}$ is the vector containing the contribution of each wave in the layer.

Other wave properties which can be useful for the physical interpretation of the material behaviour may be derived from the latter.

Wave characteristics

Phase velocity	Attenuation	Length
$V_i = \mathrm{Re}\left\{\frac{\omega}{\imath\Gamma_i}\right\}$	$\tau_i = -\omega\,\mathrm{Im}\left\{\frac{\imath\Gamma_i}{\omega}\right\}$	$\lambda_i = \mathrm{Re}\left\{\frac{2\pi}{\imath\Gamma_i}\right\}$

Quadratic indications

Analytical form	Derived form
$\mathcal{P} = \int_0^d f^*(z)g(z)\mathrm{d}z$	$\mathcal{P} = \mathbf{q}^*\left[\int_0^d \boldsymbol{\Lambda}^*(z)\,\boldsymbol{\Phi}^*\,\boldsymbol{\Xi}\{\mathcal{P}\}\,\boldsymbol{\Phi}\,\boldsymbol{\Lambda}(z)\,\mathrm{d}z\right]\mathbf{q}$

Sub-layering of the anisotropic poroelastic core and influence of the material natural axis orientation

Waves in the anisotropic poroelastic core

- The core of the studied system is composed of an anisotropic poroelastic homogeneous melamine core.
- The flow resistivity and the stiffness of the melamine has been measured and inverse estimated as fully anisotropic.

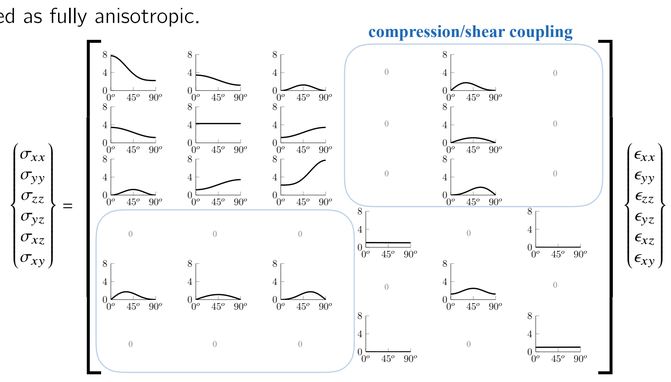


Fig. 1 - Anisotropic Hooke's law as a function of the material alignment rotation angle β[y scale = x 10^5 Pa].

- The wave properties depend on material alignment as well as the frequency.
- Strong influence of the anisotropy of the core on the wave propagation through the medium.
- The alignment of a real industrial melamine is arbitrary, and displays always a degree of shear/compression coupling.

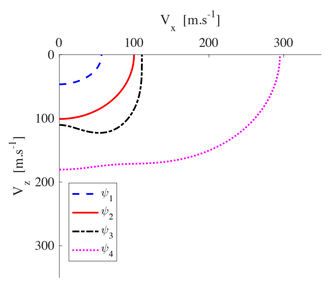


Fig. 2 - Wave velocities at *f=100* Hz

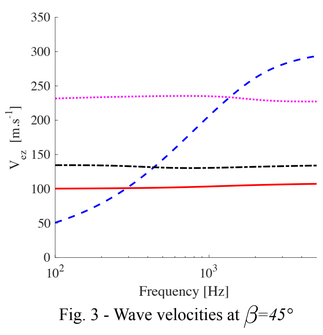


Fig. 3 - Wave velocities at β=45°

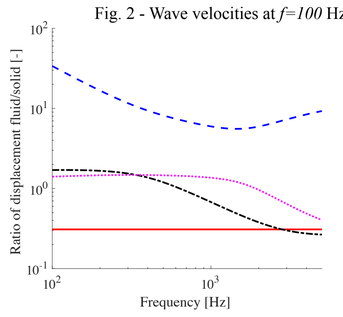
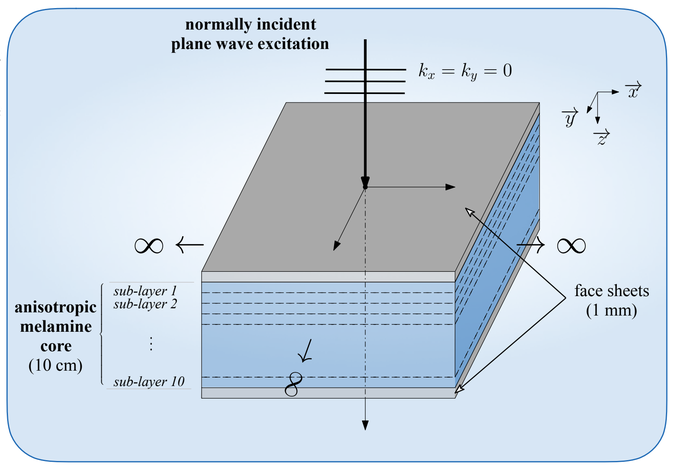


Fig. 4 - Ratio of displacement in the different phases at β=45°



Core material parameters

Anisotropic and anelastic melamine foam

Tortuosity	α_∞	1.2
Viscous charac. length	Λ	240×10^{-6} m
Thermal charac. length	Λ'	490×10^{-6} m
Porosity	ϕ	0.99
Density of frame	ρ_1	9.2 kg.m^{-3}
Fractional derivative order	$\hat{\alpha}$	0.33348
Relaxation frequency	$\hat{\beta}$	812.69×10^3 rad.s^{-1}
Anelastic contribution	$\hat{b}$	0.29620

Anisotropic flow resistivity

$$\sigma_{\text{flow}} = \begin{bmatrix}0.9727 & 0 & 0\\ & 1.0655 & 0\\ \text{sym.} & & 1.1318\end{bmatrix}\times10^4\ \text{Pa.s.m}^{-2}$$

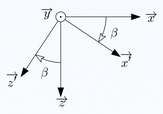
Anelastic model

$$\mathbf{H} = \mathbf{C}\left(1 + \frac{\hat{b}\left(\frac{\imath\omega}{\hat{\beta}}\right)^{\hat{\alpha}}}{1+\left(\frac{\imath\omega}{\hat{\beta}}\right)^{\hat{\alpha}}}\right)$$

Anisotropic Hooke's tensor C (x 10^5 Pa)

$$\mathbf{C} = \begin{bmatrix}7.7194 & 3.4252 & -0.0226 & 0 & 0 & 0\\ & 4.2782 & 1.1845 & 0 & 0 & 0\\ & & 2.2155 & 0 & 0 & 0\\ & & & 1.0364 & 0 & 0\\ & \text{sym.} & & & 1.2368 & 0\\ & & & & & 1.0123\end{bmatrix}\times10^5\ \text{Pa}$$

Material alignment rotation angle



Sub-layering of the core

- The core is divided into 10 sub-layers of same thickness (10 mm).
- Each sub-layer has the same nominal characteristics, and a material coordinate system independent from one another
- 6 different material coordinate system rotation profiles are studied. In each case, the material coordinate system of each sublayer is rotated. The sub-layered coordinate system rotation profiles may be seen in Fig. 5. The profiles where chosen to study the influence of gradual rotation of the material coordinate system of the sub-layers. The profiles are set by rotating the layer i={1,10} of an angle β_i (in deg.) such that:

$\beta_i = 0^o$ — $\beta_i = 45^o$ — $\beta_i = 90^o$ — $\beta_i = 10\,(i-1)$ — $\beta_i = 10\,(10-i)$ — $\beta_i = 90\,\sin\{(i-1)\,\pi/9\}$

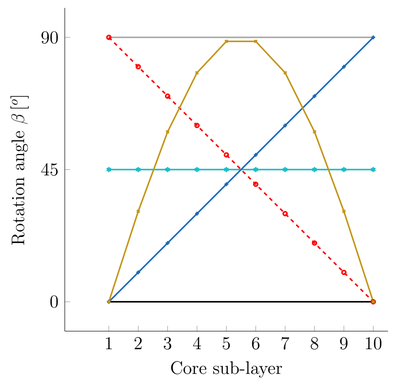


Fig. 5 - Different material coordinate system profiles

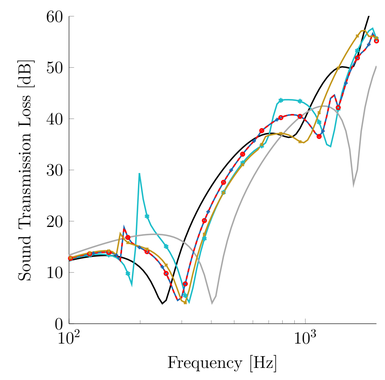


Fig. 6 - STL for the different multilayered systems with sub-layered cores

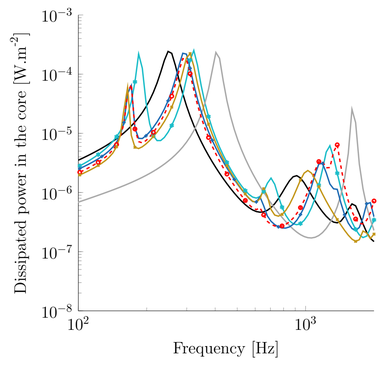


Fig. 7- Total dissipated power in the core.
The dissipated power in each sub-layer is the sum of the powers dissipated by structural, thermal and viscous means.

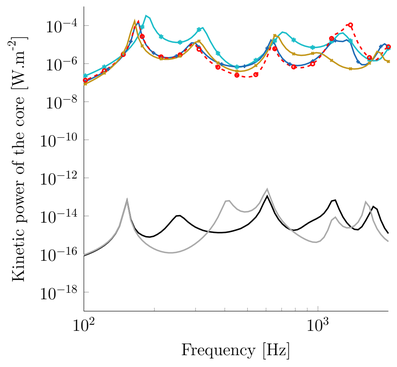


Fig. 8- Total kinetic power in the core.
The kinetic power in each sub-layer is the sum of the kinetic powers in the fluid and solid phases.

Conclusions

- The proposed method for the study of multilayered systems is based on the expansion of the dynamic solution as a superposition of plane waves.
- The formulation relies on a state-space representation in terms of physical field variables, and directly provides the characteristics of the waves in the different layers of the structure.
- The state-space representation requires the computation of the state matrix, characterising the dynamic state of each material layer in a system.
- Using the proposed method, it was found that, for an anisotropic industrial melamine foam, there is a strong influence of the inherent anisotropy of the material in the wave propagation through the medium.
- Having established the effects of the anisotropy on the wave propagation through the medium, the influence of the anisotropy of poroelastic cores on the dynamic behaviour of multilayered systems has been evaluated.
- The relative alignment of the porous sub-layers has important influence on the acoustic transmission through the system, creating, damping or shifting resonances in the multilayered system.
- By sub-layering the system, the mechanical and acoustic response may be tuned without increasing the weight of the structure, shedding light into a potentially optimisable problem.
- The results show a potential benefit of functionally graded relative alignment of anisotropic poroelastic cores on the behaviour of multilayered systems.

Bibliography

[1] O. Dazel *et al.*, A stable method to model the acoustic response of multilayered structures, J. Appl. Phys. 113 (8), p. 83506, 2013.

[2] C. Van der Kelen and P. Göransson, Identification of the full anisotropic flow resistivity tensor for multiple glass wool and melamine foam samples, J. Acoust. Soc. Am. 134(6), p.4659, 2013.

[3] J. Cuenca, *et al.*, A general methodology for inverse estimation of the elastic and anelastic properties of anisotropic open-cell porous materials with application to a melamine foam, J. Appl. Phys. 115(8), p.84904, 2014.

[4] C. Van der Kelen, *et al.*, Property modelling A method for the inverse estimation of the static elastic compressional moduli of anisotropic poroelastic foams e With application to a melamine foam, Pol. Test. 43, p.123-130, 2015.

[5] C. Van der Kelen *et al.*,A method for characterisation of the static elastic properties of the porous frame of orthotropic open-cell foams, Int. J. Eng. Sc. 86, p.44-59, 2015.

[6] J. P. Parra Martinez, *et al.*, *Acoustic analysis of anisotropic poroelastic multilayered systems*, J. Appl. Phys. 119(8), 2016.

[7] J. P. Parra Martinez *et al.*, Derivation of the state matrix for dynamic analysis of linear homogeneous media, J. Acoust. Soc. Am. 140(2), EL218-EL220, 2016.