Johnson-Champoux-Allard Model in Thermoacoustic

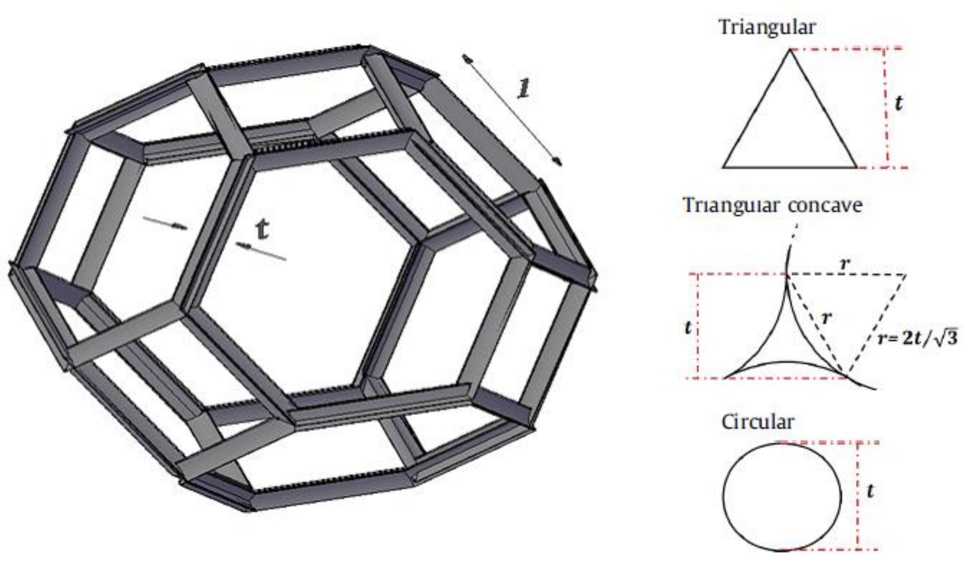
Dragonetti R., Napolitano M., Romano R.

University of Naples, Federico II – P.le Tecchio 80, 80125 Naples
Department of Industrial Engineering - Laboratory of Acoustics

The thermal and viscous interactions that occur between the solid and working fluid inside the stack/regenerator are studied by using the spatially averaged viscous and thermal functions f_v and f_k. They can be written using the **Johnson-Champoux-Allard Model**. It requires the knowledge of five parameters:

- **ϕ** porosity
- $\boldsymbol{\alpha_\infty}$ tortuosity
- $\boldsymbol{\Lambda_t}$ thermal characteristic length
- $\boldsymbol{\Lambda_v}$ viscous characteristic length
- **σ** resistivity

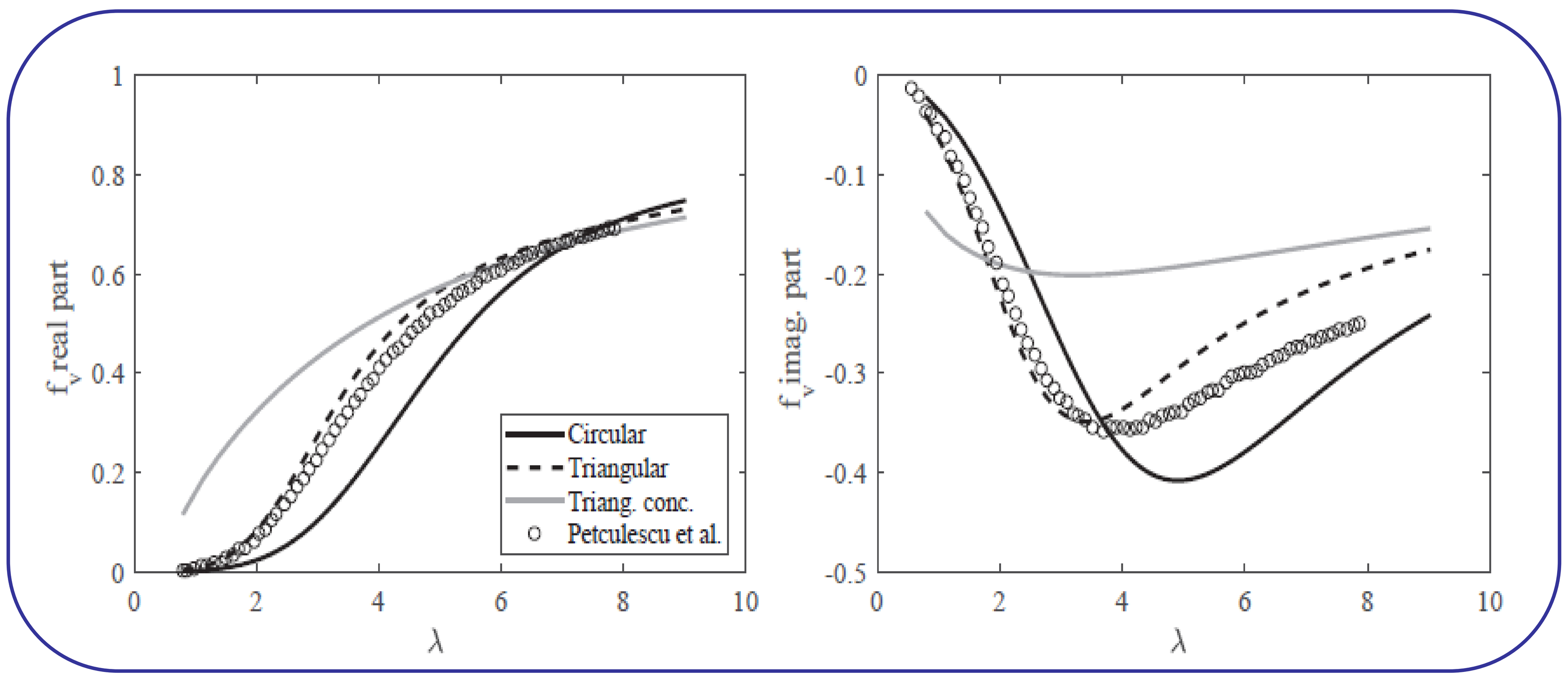
$$f_v = 1 - \frac{1}{\alpha_\infty\left(1 + \frac{\phi\sigma}{j\omega\rho_m\alpha_\infty}\sqrt{1 + j\frac{4\alpha_\infty^2\eta\rho_m\omega}{\phi^2\sigma^2\Lambda_v^2}}\right)} \qquad f_k = 1 - \frac{1}{1 + \frac{8\eta}{j\Lambda_t^2 P_r\omega\rho_m}\sqrt{1 + j\frac{\Lambda_t^2\rho_m P_r\omega}{16\eta}}}$$

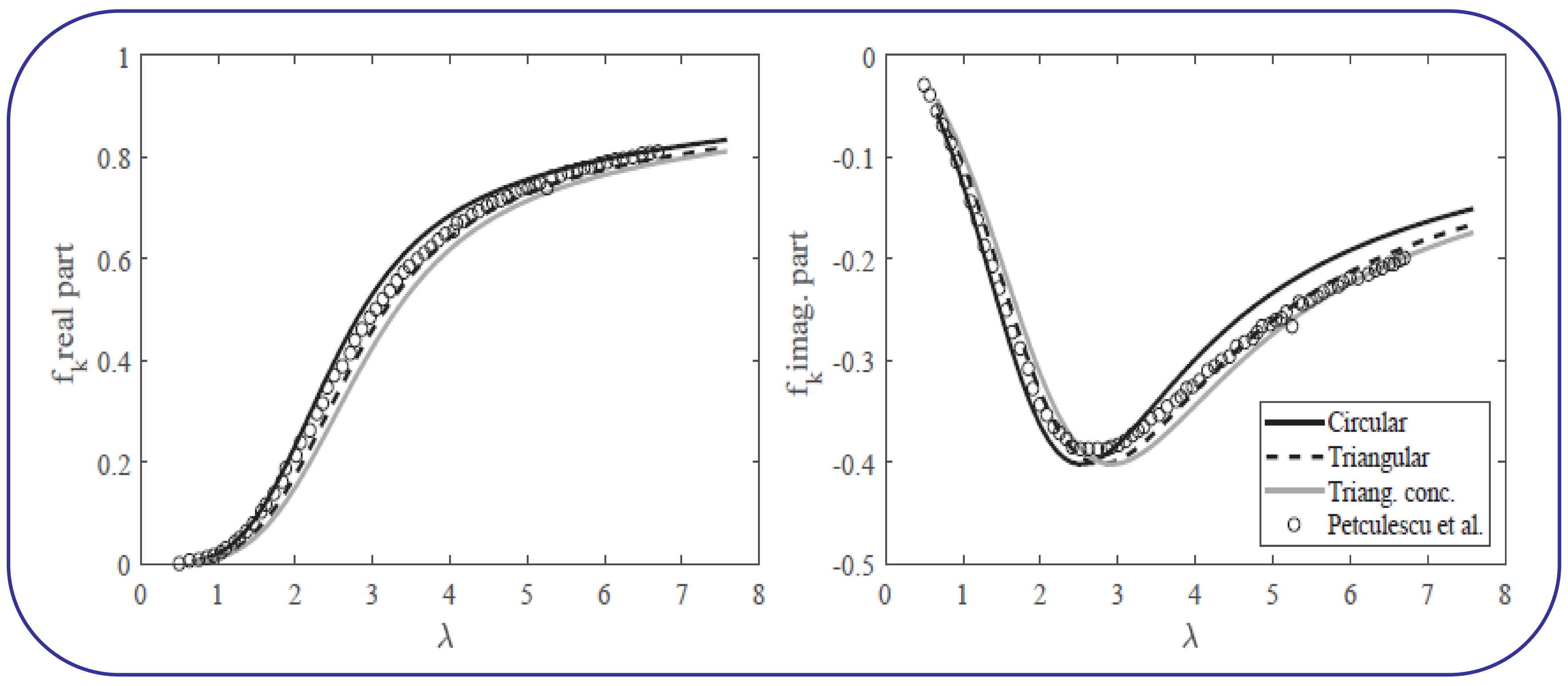


Making use of the general functions f_v and f_k it is possible to investigate the thermoacoustic performance of the open-cell materials foams. This kind of material is modelled using as basic unit, the Kelvin tetrakaidecahedron cell characterized by strut length l and thickness t.
The characteristic radius of the cell is $R = {}^{1}/_{ppi}$.

A comparison has been conducted with the experimental results reported by *Petculescu et al.* [1]. It is referred to a RVC foam that has 20 pores per inch, $l = 730\ \mu m$, $t = 240\ \mu m$ and $R_w = 1$.

$\lambda = \sqrt{2R}\,\delta_v$ $\qquad\qquad$ $\lambda = \sqrt{2R}\,\delta_k$

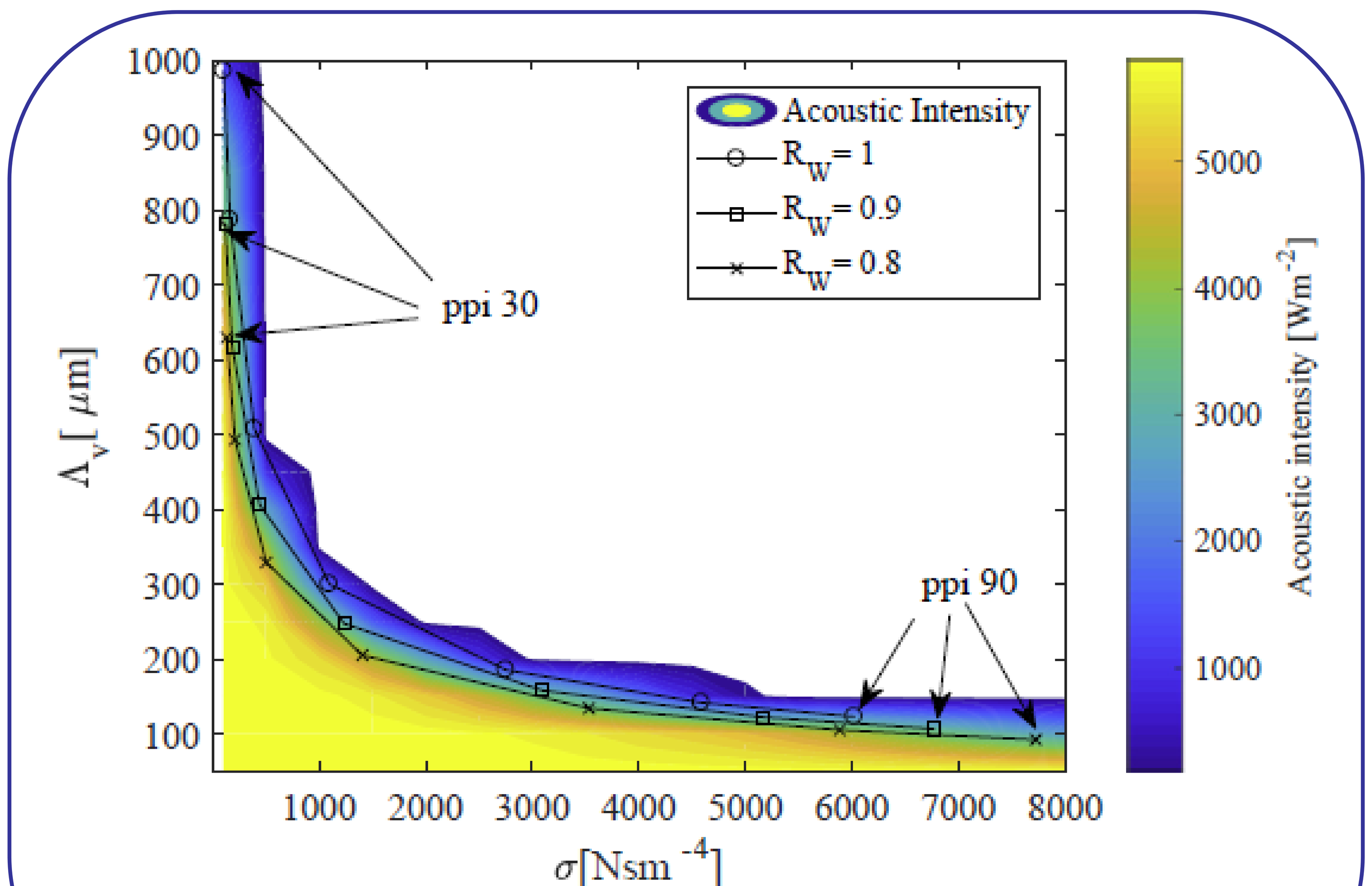




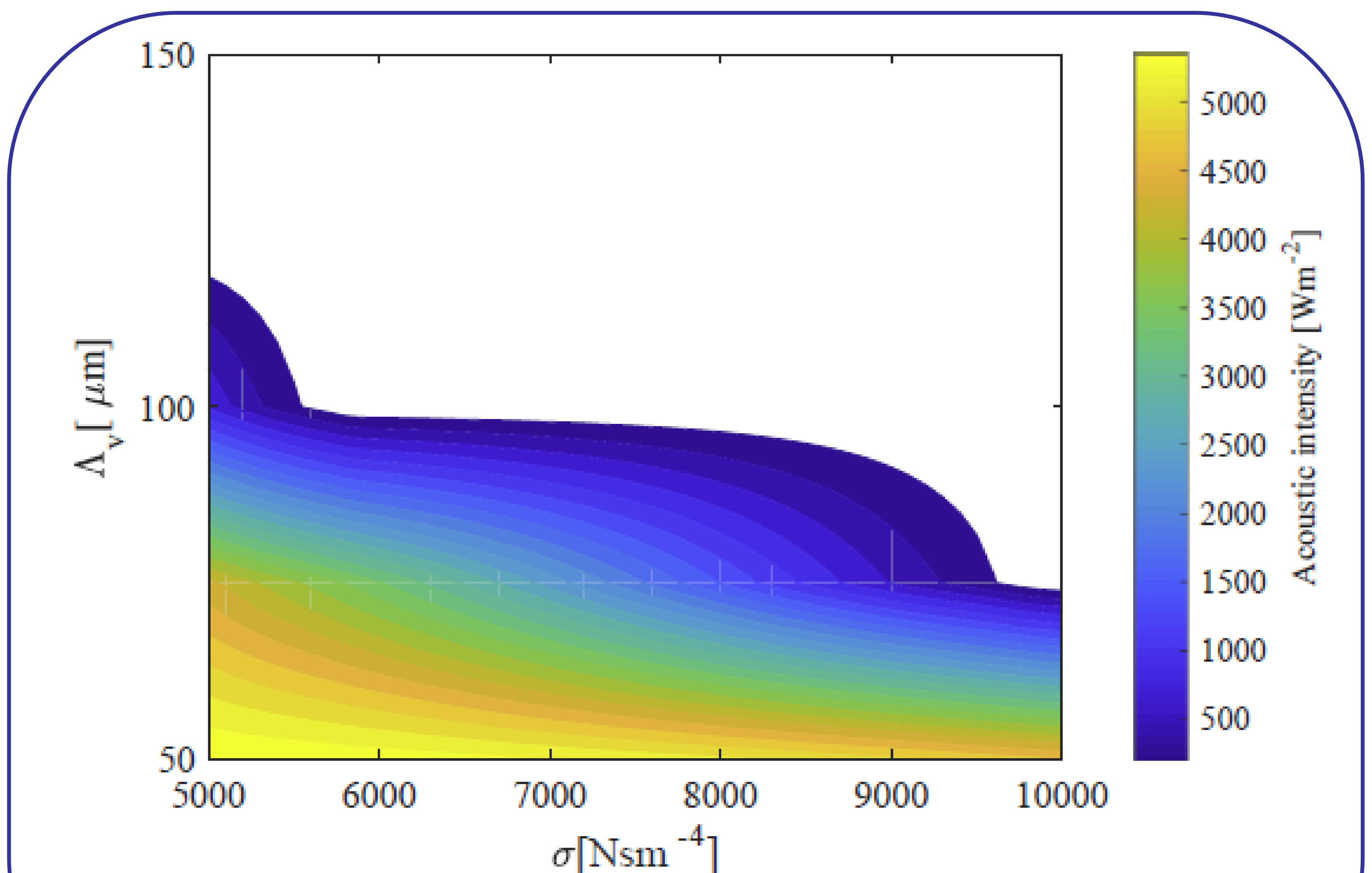
To analyse the influence of the non acoustic parameters of porous material on the performance of the thermoacoustic devices, a optimization procedure has been developed. It provides the maximum available **Acoustic Intensity**.

$$\frac{d\dot{E}}{A_{fluid}dx} = -\frac{1}{2}\omega\rho_m\frac{-Im(f_v)}{|1-f_v|^2}|\langle u_1\rangle|^2 - \frac{1}{2}\frac{-Im(f_k)\omega(\gamma-1)}{\gamma p_m}|p_1|^2 + \frac{1}{2}Re\left[\frac{(f_k-f_v)}{(1-f_v)(1-P_r)}\frac{dT_m}{dx}\frac{1}{T_m}\tilde{p}_1\langle u_1\rangle\right]$$

Results are referred to the simple case of a standing-wave engine with a stack made up of RVC foam.



$\alpha_\infty = 1.05;\ \phi = 97\%;\ \Lambda_t = 1.8\Lambda_v$



$\alpha_\infty = 1;\ \phi = 99\%;\ \Lambda_t = 2\Lambda_v$

The optimum can be achieved for low values of air flow resistivity and viscous characteristic length. Unfortunately, for a fully reticulated open-cell foam it is impossible to get both these conditions. Results suggest that this problem can be theoretically sidestepped by using a partially reticulated foam. Open-cell materials with $R_w = 1 - 0.9$ provide results that are close to those obtained with traditional stack while reducing the parameter R_w allows to obtain more energy and therefore the latter can improve thermoacoustic performances.

[1] Petculescu A., Wilen LA., *Lumped-element technique for the measurement of complex density*, J. Acoustic Soc. Am. 2001, 110, 1950-7