Thermoacoustics: an overview

Gaëlle POIGNAND
Laboratoire d'Acoustique de l'Université du Mans
UMR CNRS 6613

Thermoacoustics TEAM

G. Poignand	*Research Engineer*
G. Penelet	*Assistant Professor*
P. Lotton	*Senior Researcher*
V. Zorgnotti	*PhD Student*
L. Gong	*PhD Student*

Thermoacoustics: an overview

I. What is thermoacoustics ?

II. Linear theory of thermoacoustic

III. Focus on

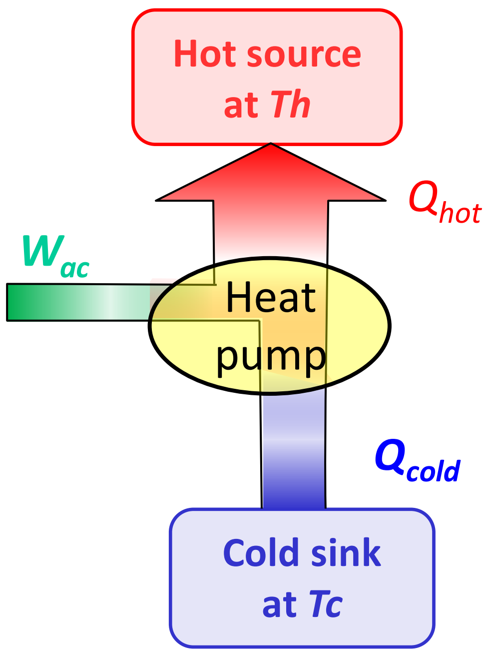
- Transfer matrix measurement
- Active tuning of acoustic oscillations

What is thermoacoustics ?

- **Thermoacoustic** : study of the interactions between the acoustic and the thermal wave in a porous material
- **Thermoacoustic systems** ≡ thermal machines

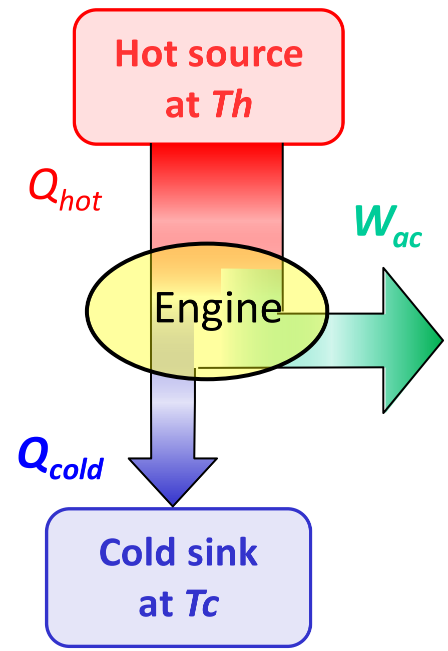
Thermoacoustic heat pump

transfers heat from low to high temperature using W_{ac} the acoustic work



Thermoacoustic prime mover

converts heat into W_{ac} acoustic work



What is thermoacoustics ?

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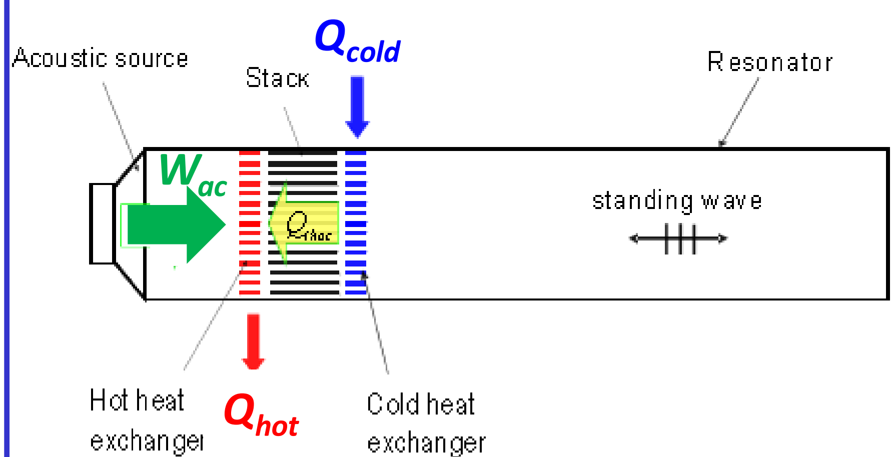
- **Thermoacoustic systems** ≡ thermal machines

Thermoacoustic heat pump

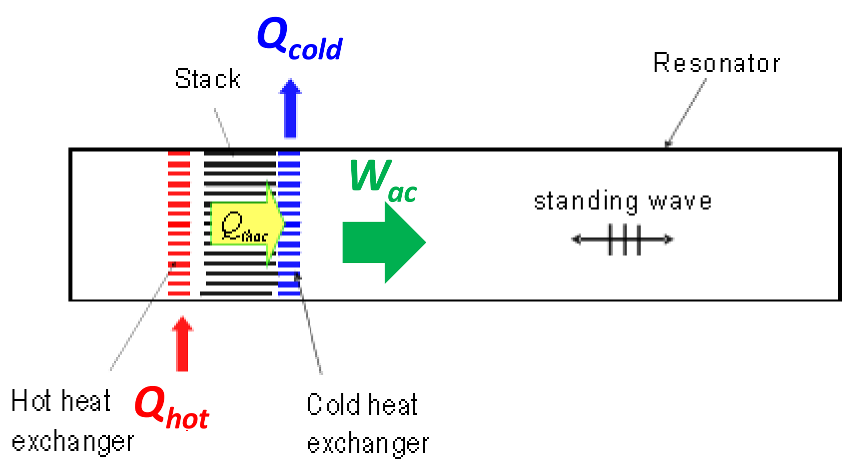
transfers heat from low to high temperature using W_{ac} the acoustic work



⇨ Thermoacoustic heat flux generation

Thermoacoustic prime mover

converts heat into W_{ac} acoustic work

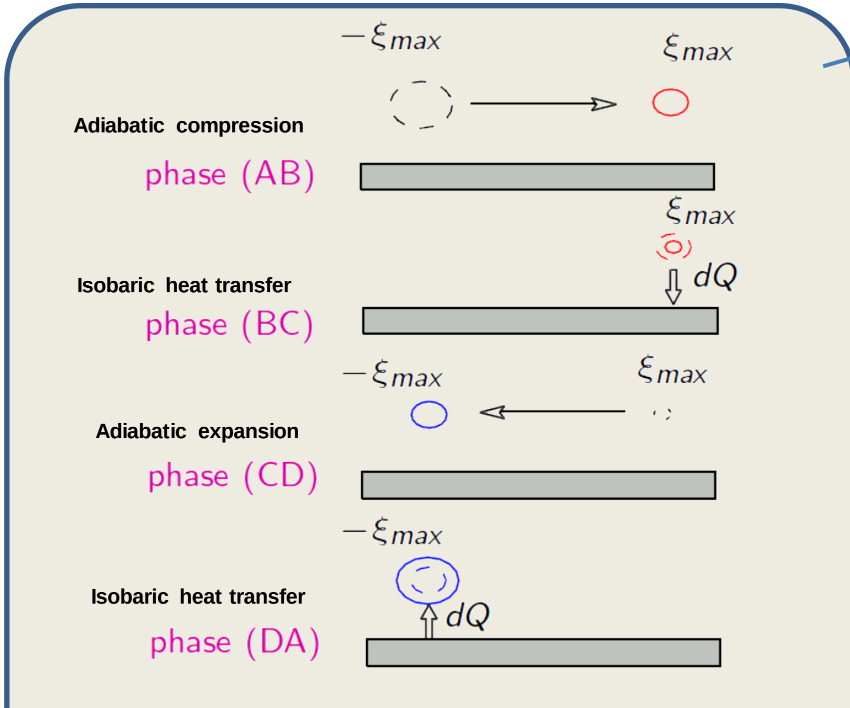


⇨ Acoustic wave generation (at the frequency of the most unstable mode)

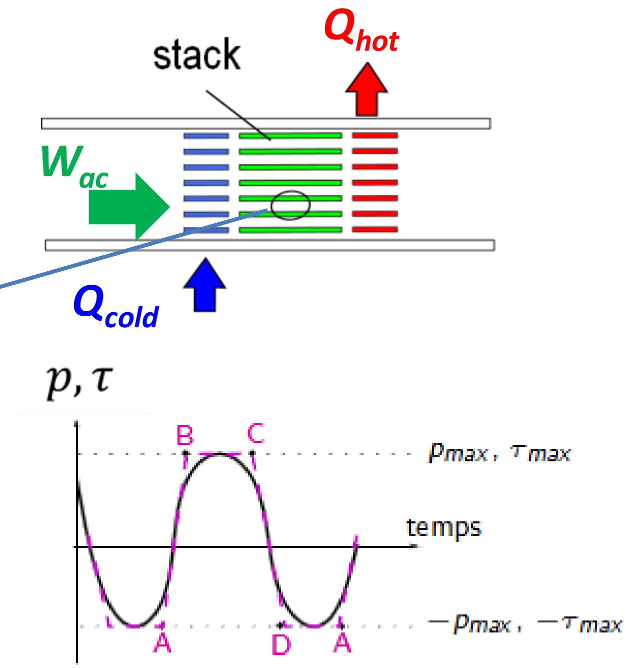
Thermoacoustic effect

Standing wave thermoacoustic heat-pump

- p, τ are in phase, p and v are in quadrature
- $\dfrac{dT}{dx} \gg \dfrac{\tau}{\xi}$



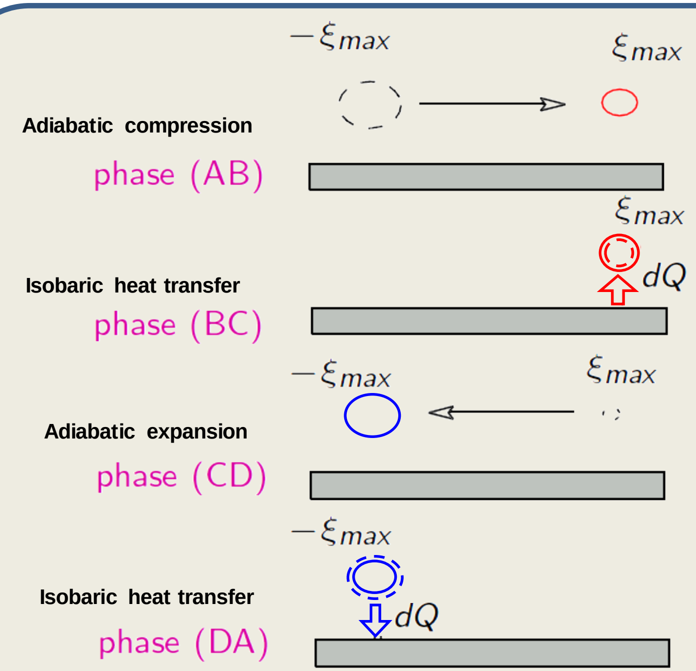
Fluid parcels as it oscillate along a plate during an acoustic period (Lagrangian point of view)

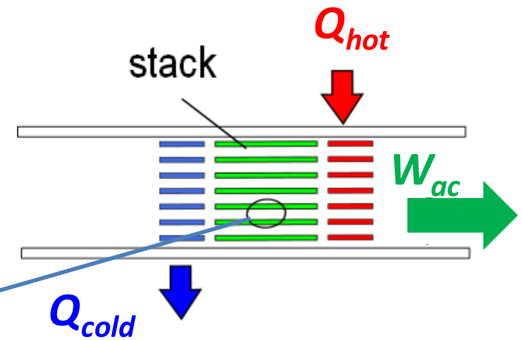


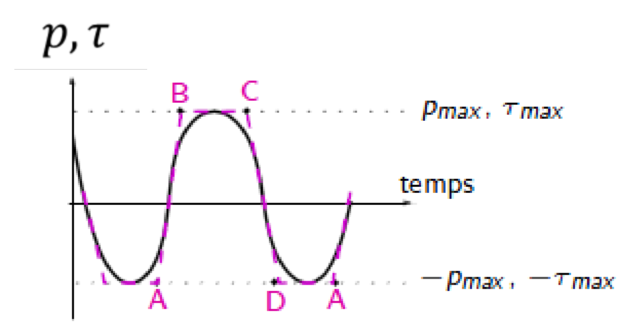
Thermoacoustic effect

Standing wave thermoacoustic engine

- p, τ are in phase, p and v are in quadrature
- **Temperature gradient** imposed by the heat exchangers and $\xi_{max}\dfrac{dT}{dx} \gg \tau_{max}$



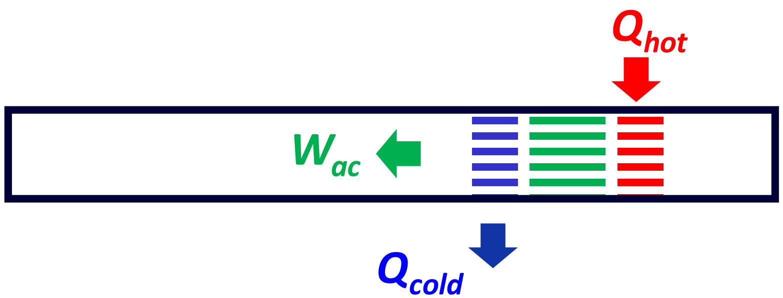




"If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged."
J. W. Strutt (Lord Rayleigh)

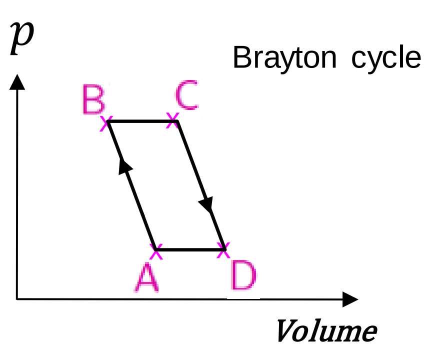
Thermoacoustic effect

Standing wave thermoacoustic engine



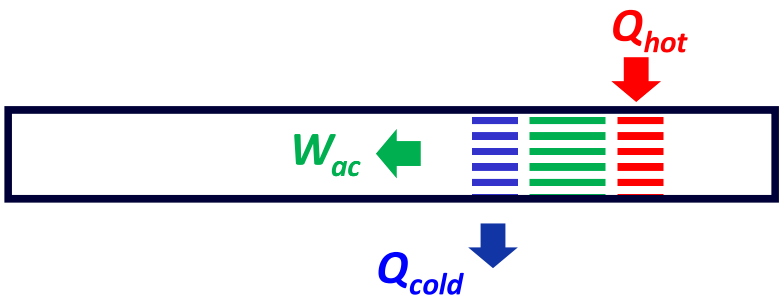
p and v are in quadrature

gas parcels should be at about δ_κ from the wall
δ_κ: the thermal boundary layer thickness
thermal contact is imperfect
Stack: pore's radius $\sim \delta_\kappa$



Thermoacoustic effect

Standing wave thermoacoustic engine



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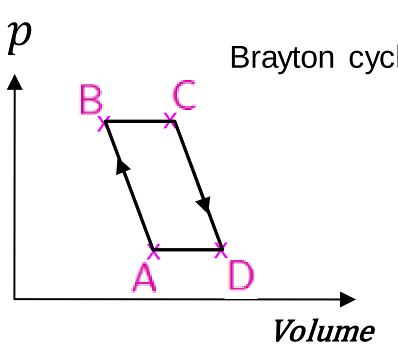
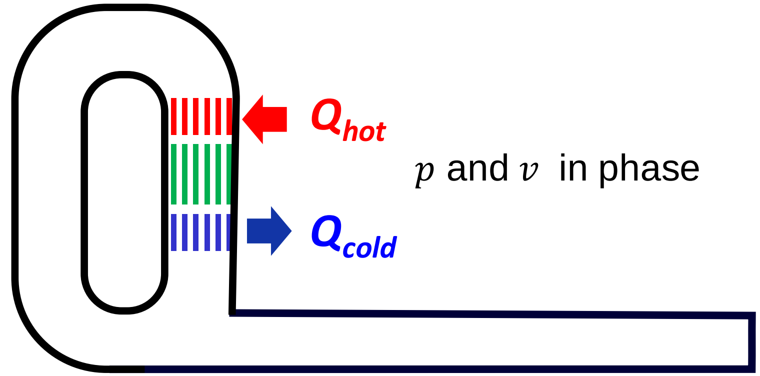
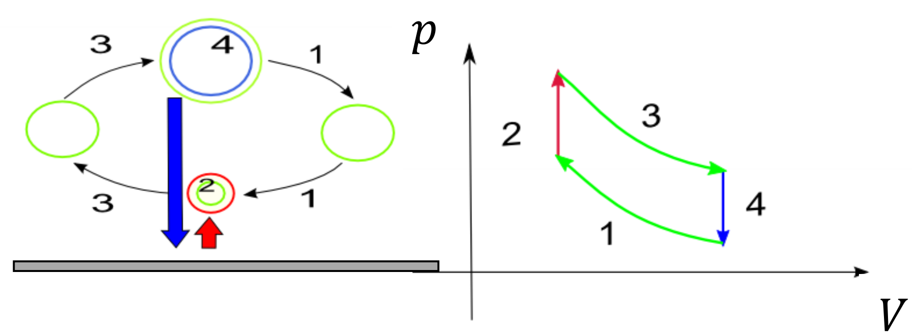


Travelling wave thermoacoustic engine



p and v in phase

gas parcels should be at less than δ_κ from the wall
isothermal contact
Regenerator: pore's radius $\ll \delta_\kappa$

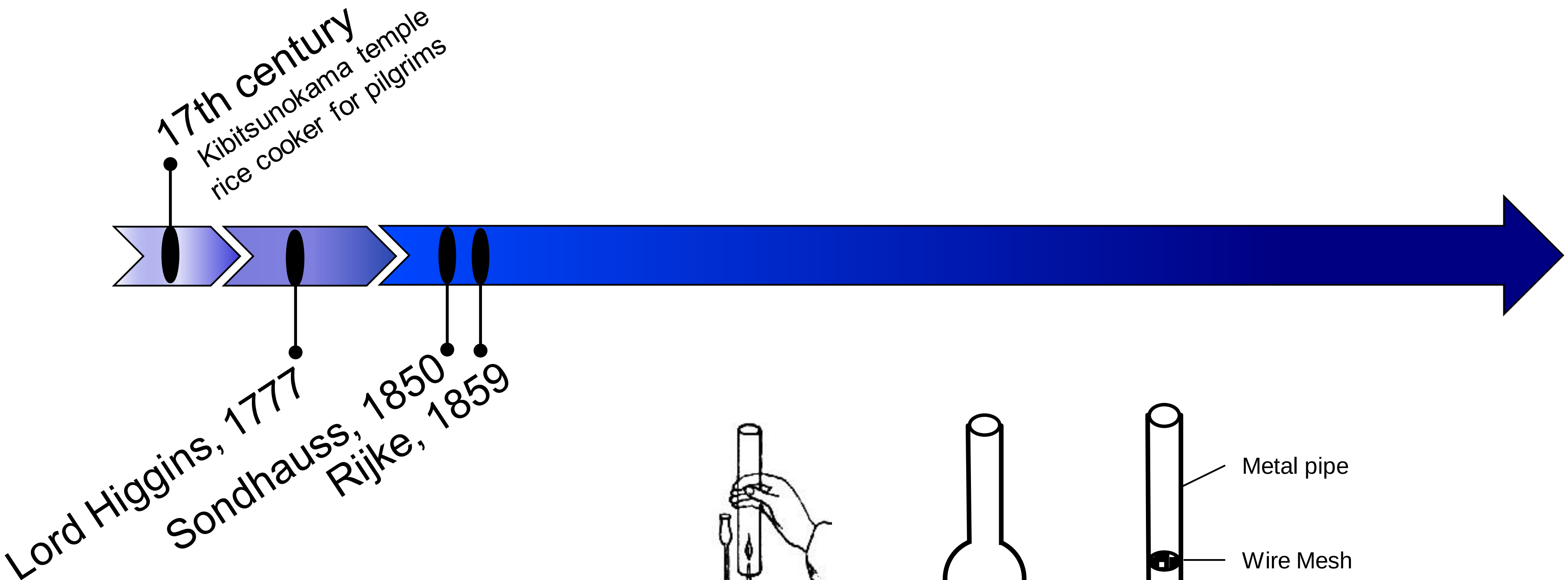


1 : Isothermal compression
2 : Isochoric heating
3 : Isothermal expansion
4 : Isochoric cooling

Stirling cycle

More efficient

Thermoacoustic history



first observation of conversion
of heat into acoustic power

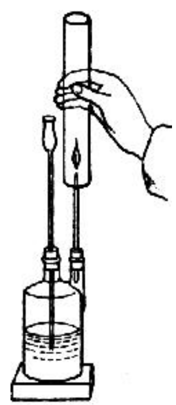
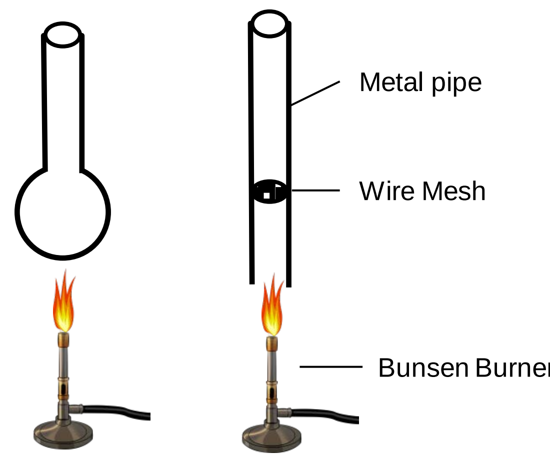


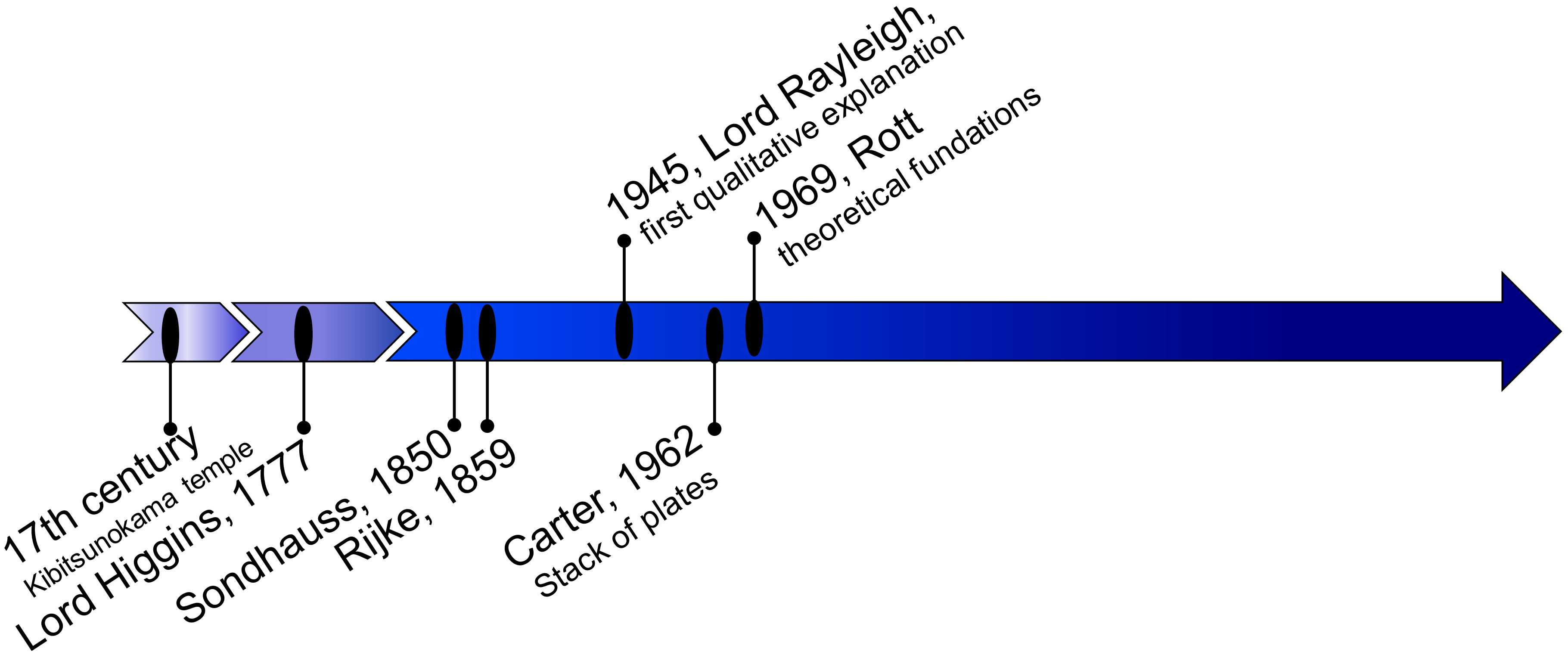
Fig. 344
Singing flame
(hydrogen)



Sondhauss pipe **Rijke pipe**

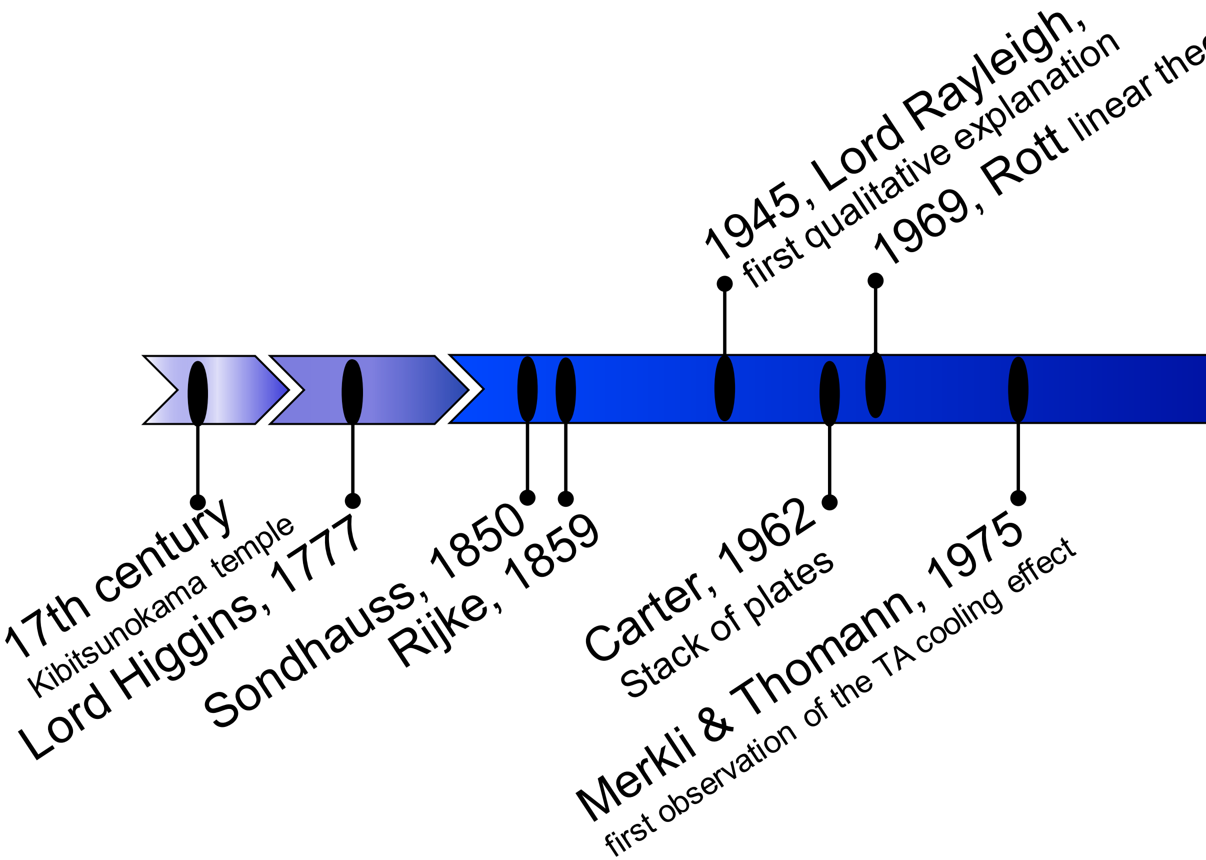
B. Higgins, Nicholson's J. London (1802), p. 130
Sondhauss, Pogendorff's Annalen der Physik und Chemie, vol. 79, pages 1-34,1850.
Rijke, Philosophical Magazine, vol. 17, pages 419-422,1859.

Thermoacoustic history



Rayleigh, J.W.S.,The theory of sound. Dover, New York, 2nd edition, 1945
Carter, R.L., White, M. and Steele, A.M. Private Communication of Atomics International Division of North American Aviation,1962.
Rott, N., Damped and thermally driven acoustic oscillations in wide and narrow tubes. Z. Angew. Math. Phys. 20, 230−243, 1969.

Thermoacoustic history



Thermoacoustic effects in a resonance tube

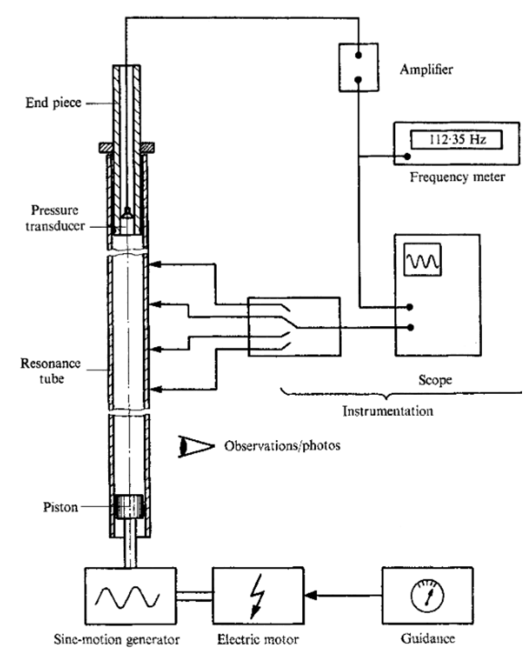
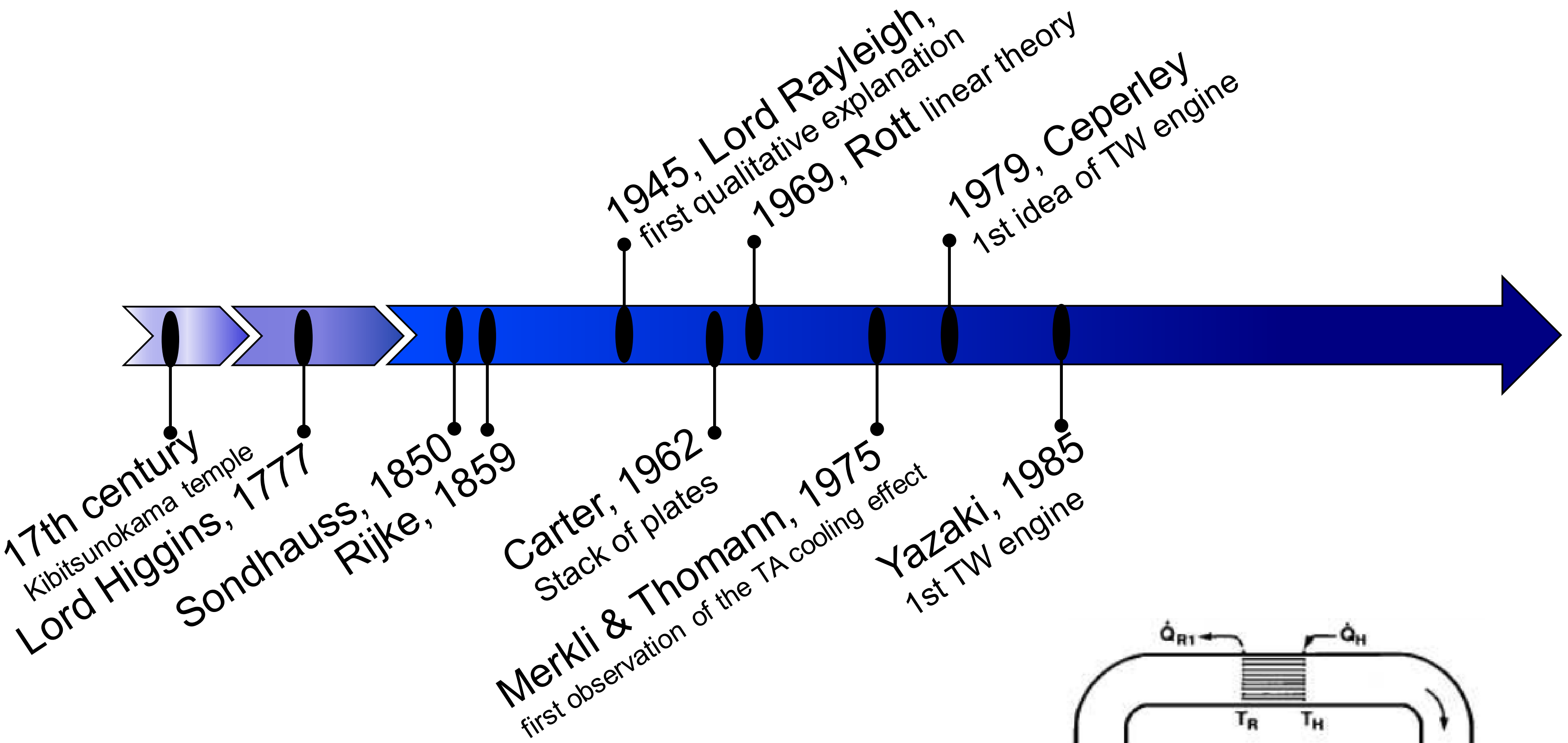
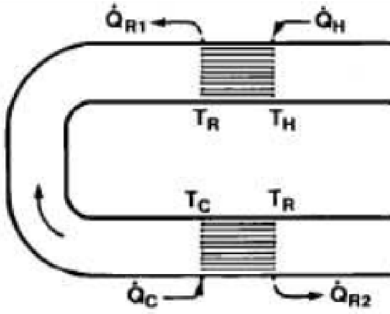


FIGURE 3. Experimental arrangement.

Merkli, P., & Thomann, H. (1975). Thermoacoustic effects in a resonance tube. Journal of Fluid Mechanics, 70(1), 161-177.

Thermoacoustic history

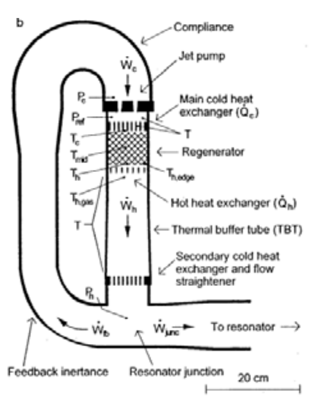


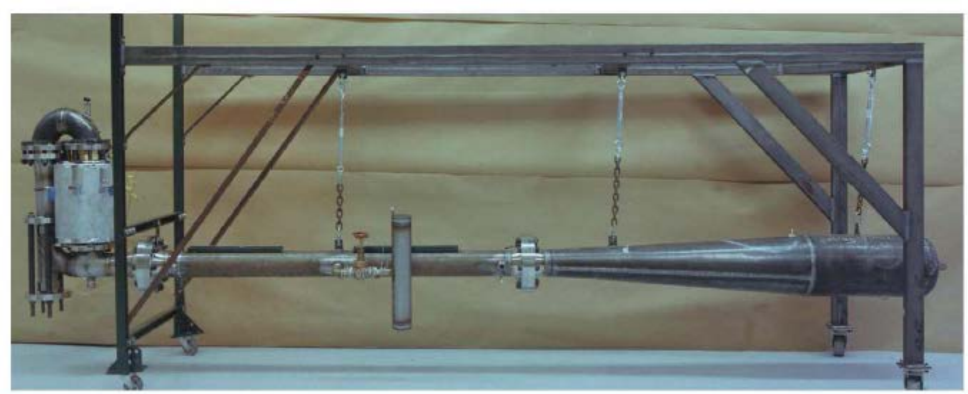


Ceperley, P.H., 1979. A pistonless Stirling engine − The traveling wave heat engine. JASA 66, 1508.
Yazaki, T., Iwata, A., Maekawa, T., Tominaga, A., 1998. Traveling Wave Thermoacoustic Engine in a Looped Tube. Phys. Rev. Lett. 81, 3128.

Thermoacoustic history

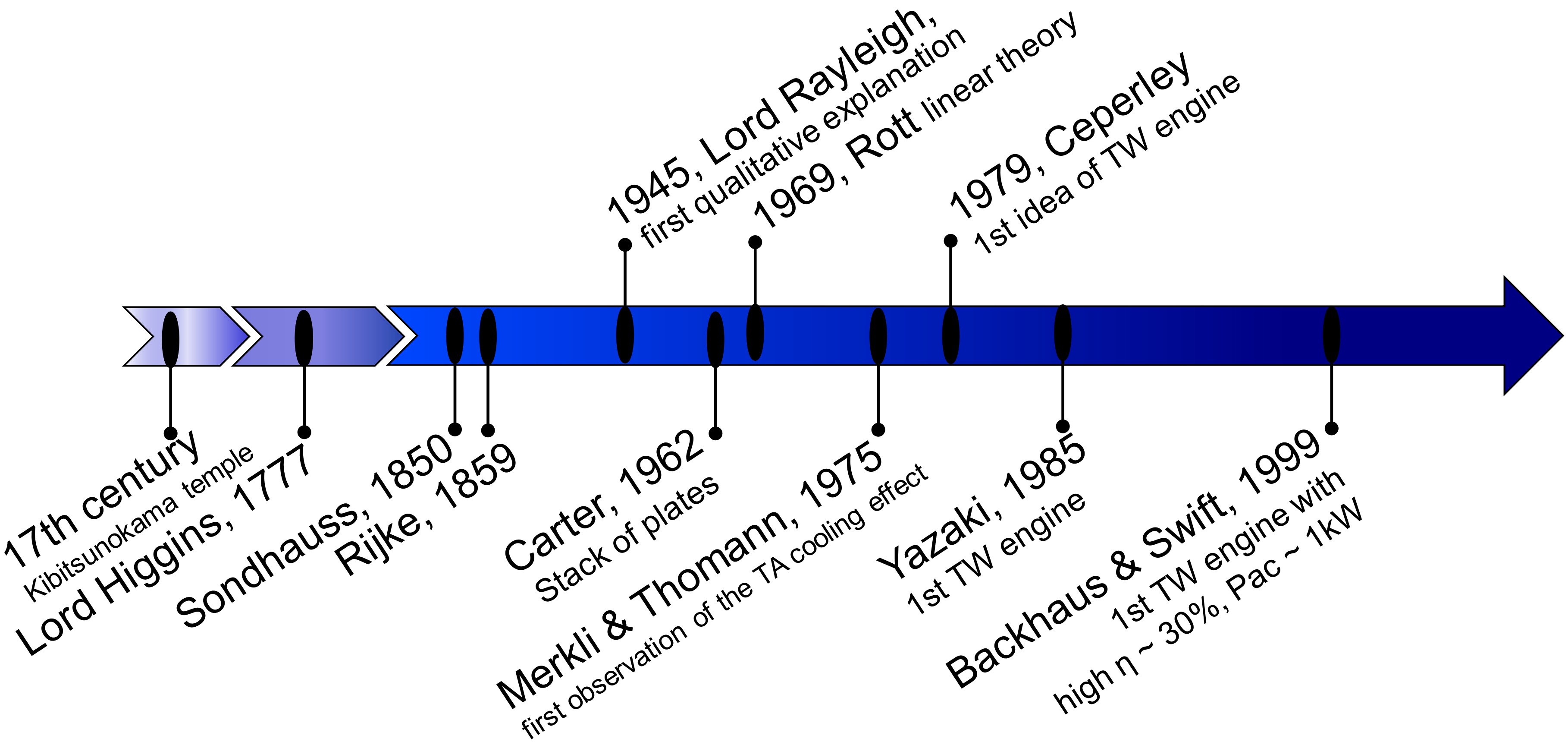
- 17th century, Kibitsunokama temple
- Lord Higgins, 1777
- Sondhauss, 1850
- Rijke, 1859
- 1945, Lord Rayleigh, first qualitative explanation
- Carter, 1962, Stack of plates
- 1969, Rott linear theory
- Merkli & Thomann, 1975, first observation of the TA cooling effect
- 1979, Ceperley, 1st idea of TW engine
- Yazaki, 1985, 1st TW engine
- Backhaus & Swift, 1999, 1st TW engine with high η ~ 30%, Pac ~ 1kW





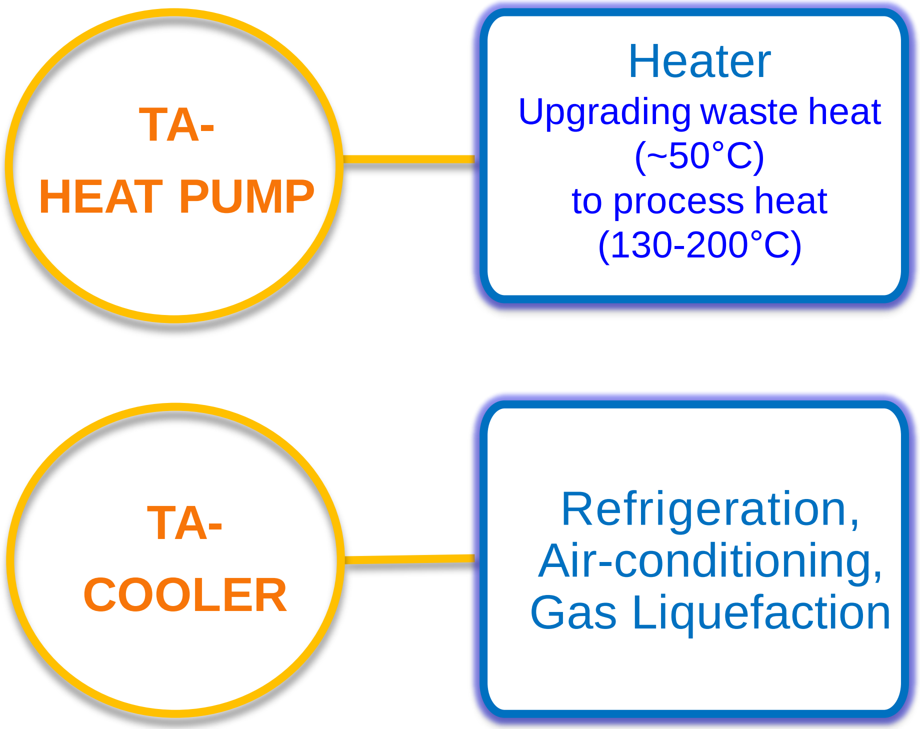
S. Backhaus and G. W. Swift, Nature 399, 335 (1999).

Thermoacoustic history

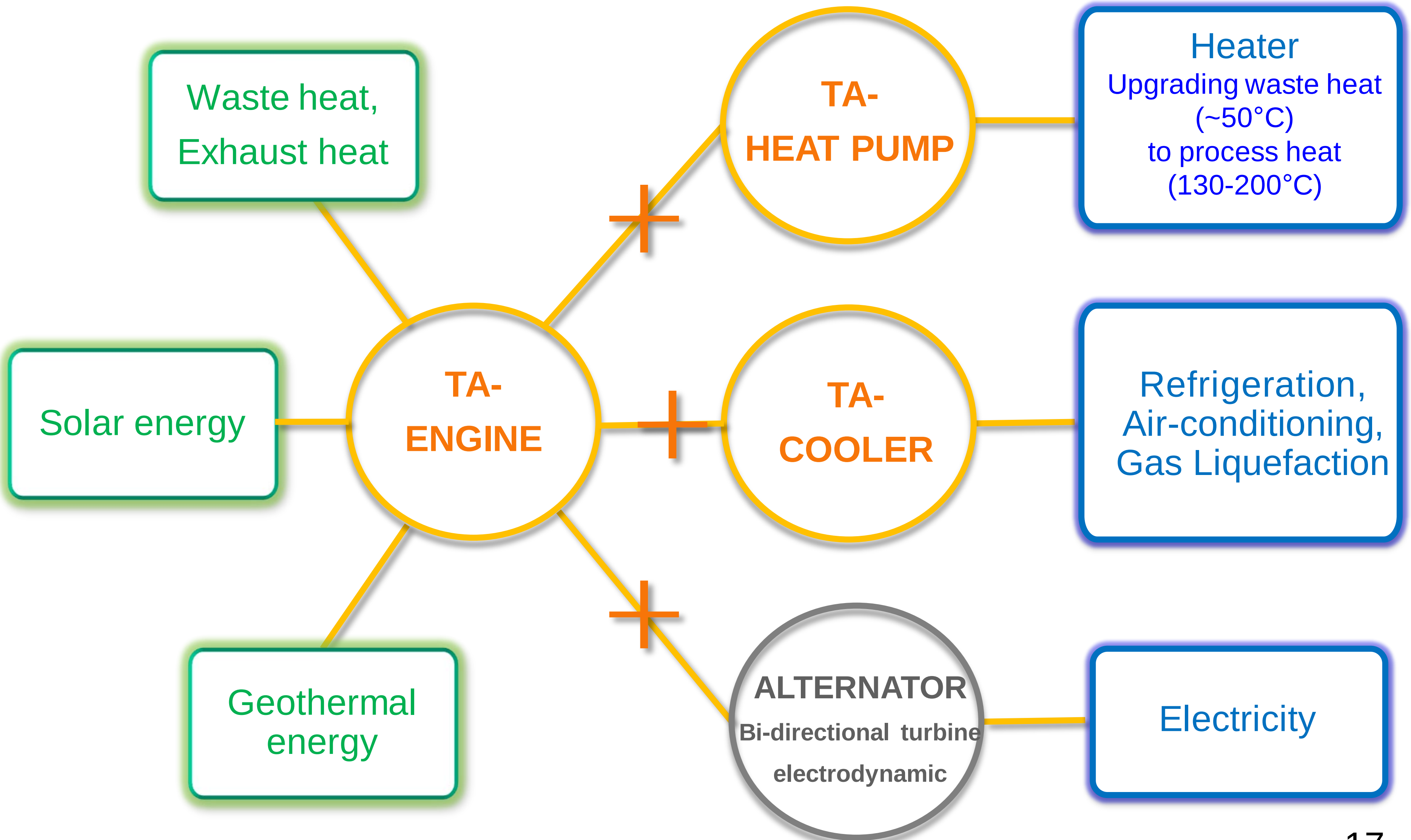


Various prototypes of thermoacoustic engines or heat pumps

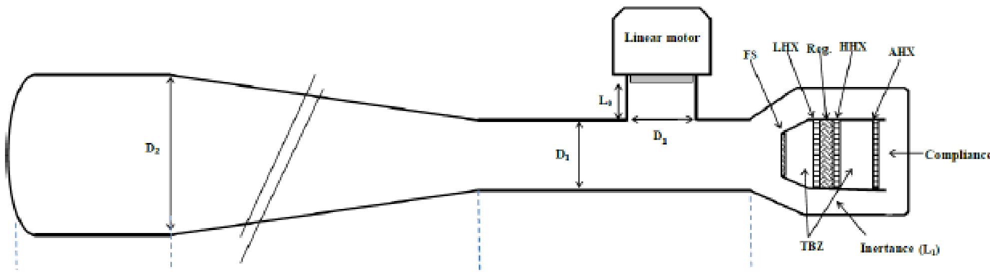
Applications

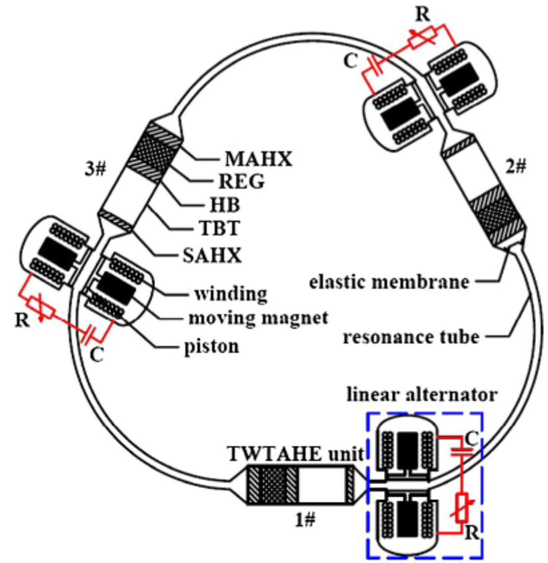


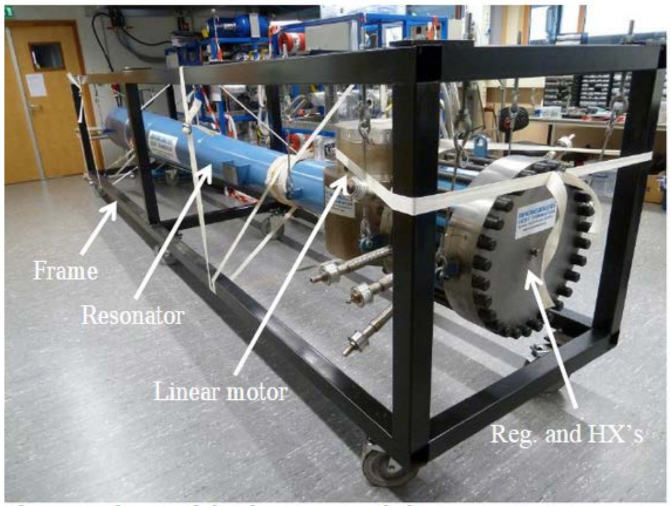
Applications

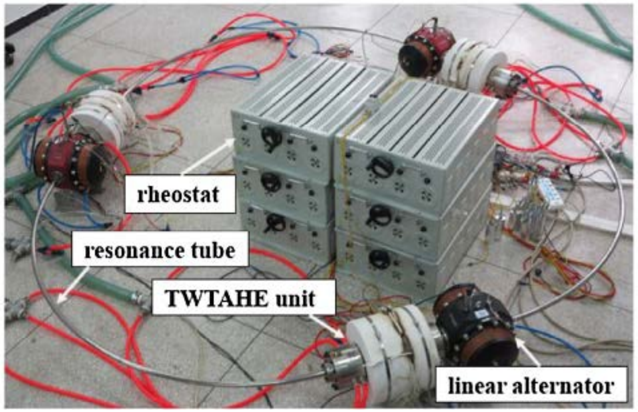


Examples of TA systems









2015 [1]: Heat pump for domestic applications

He, 50 bars,

Th≈ 109 °C, Qhot ≈ 3 kW, COP ≈ 3, COPr ≈ 42 %

2017[2]: TA electric generator

He, 60 bars,

$\Delta T \approx 630$ K, $P_{el} \approx 3{,}5$ kW , $\eta_{tot} \approx 18{,}4$ %

[1] Tijani, M. E. H., et J. A. Lycklama à Nijeholt 2015.
[2] Bi, Tianjiao, Zhanghua Wu, Limin Zhang, Guoyao Yu, Ercang Luo, et Wei Dai. *Applied Energy*, 185, 2017

Thermoacoustic systems

Advantages

- ✓ Environmental friendly ⇨ no environment harmful fluids (inert gas)
- ✓ Construction is simple, no or few solid moving parts: robust and reliable
- ✓ High potential efficiency (max : η = 32%, η_c =49%)

Drawback

- ✓ Power limited (~10 kW)
- ✓ Efficiency could still be improved due to
  - Complex process which are not fully understood:
    - Heat exchanger in oscillating flow
    - Heat transfer from the stack to the heat exchanger
  - Various losses:
    - Acoustic streaming
    - Non linear temperature oscillation harmonics

⇨ To date, industrial development limited

1. Tijani, Journal of Applied Physics, 2011

Thermoacoustics: an overview

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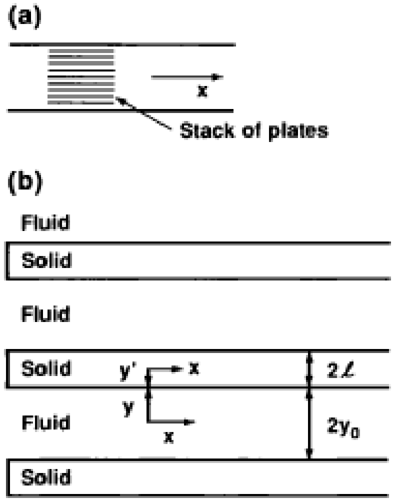
- Active tuning of acoustic oscillations

Linear theory

Plane wave propagation through a viscous and heat-conducting gas along a duct submitted to a temperature gradient.



Governing equation:

- momentum equation
- continuity equation
- energy equation for the fluid
- thermodynamic state equation

+ boundary conditions: $\tau|_{y\,=\,\pm y0}=0$ and $v_x|_{y\,=\,\pm y0}=0$

Main assumptions:

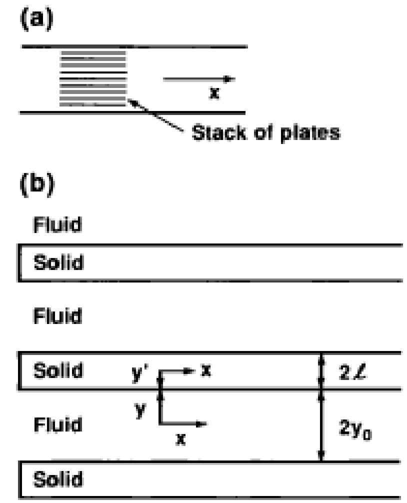
- Ideal gas, with no mean flow,
- $\rho_s C_s \gg \rho_0 C_p$ and $\lambda_s \gg \lambda$
- Low amplitude:

$$P = P_0(x) + \; p(x,t)$$
$$\rho = \; \rho_0(x) + \rho'\,(x,y,t)$$
$$T = \; T_0(x) \; + \; \tau(x,y,t)$$
$$\vec{v}(x,y\,,t) = vx(x,y,t)\overrightarrow{e_x} + vy(x,y,t)\overrightarrow{e_y} \ll c_0$$
$$S = \; S_0(x) \; + \; s(x,y,t)$$

- Typical wavelength $\gg R = 2y_0$
  - Plane wave $p(x,y,t) = p(x,t)$
  - Boundary layer approximation $|\partial_y \zeta| \gg |\partial_x \zeta|$ $\zeta = p, \rho', \tau, v, s$(transverse variation higher than longitudinal)

Linear theory

All acoustic variables can be expressed in terms of p and $\frac{\partial p}{\partial x}$



$$v_x(x,y) = \frac{i}{w\rho_m(x)}\frac{\partial p}{\partial x}[1 - Fv(y)]$$

$$\tau = \frac{p}{\rho_{m(x)}C_p}[1 - F_\kappa(y)] - \frac{1}{\rho_m w^2}\frac{\partial p}{\partial x}\frac{\partial T_m}{\partial x}\left[1 - \frac{\sigma F_v(y) - F_\kappa(y)}{\sigma - 1}\right]$$

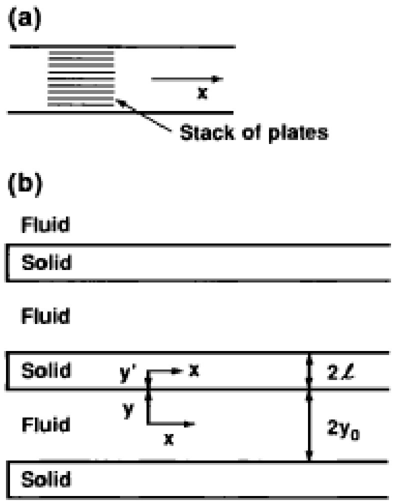
$$\rho' = \frac{1}{w^2}\left[1 - \frac{\sigma}{\sigma - 1}F_v + \frac{1}{\sigma - 1}F_\kappa\right]\frac{\partial_x T_m}{T_m}\frac{\partial p}{\partial x} + \frac{1}{{c_0}^2}[1 + (\gamma - 1)F_\kappa]p$$

$$s = \frac{p}{\rho_m T_m}F_\kappa - \frac{C_p}{\rho_m w^2}\frac{\partial p}{\partial x}\frac{\partial_x T_m}{T_m}\left[1 - \frac{\sigma F_v(y) - F_\kappa(y)}{\sigma - 1}\right]$$

with $F_v(y) = \dfrac{\cosh\left(\frac{(1+i)y}{\delta_v}\right)}{\cosh\left(\frac{(1+i)y_0}{\delta_v}\right)}$ and $F_\kappa(y) = \dfrac{\cosh\left(\frac{(1+i)y}{\delta_\kappa}\right)}{\cosh\left(\frac{(1+i)y_0}{\delta_\kappa}\right)}$

Linear theory

All acoustic variables can be expressed in terms of p and $\frac{\partial p}{\partial x}$
+ averaged over the duct's cross-section



$$\langle v_x \rangle = \frac{i}{w\rho_m(x)}\frac{\partial p}{\partial x}[1 - fv]$$

$$\langle \tau \rangle = \frac{p}{\rho_m(x)C_p}[1 - f_\kappa] - \frac{1}{\rho_m w^2}\frac{\partial p}{\partial x}\frac{\partial T_m}{\partial x}\left[1 - \frac{\sigma f_v - f_\kappa}{\sigma - 1}\right]$$

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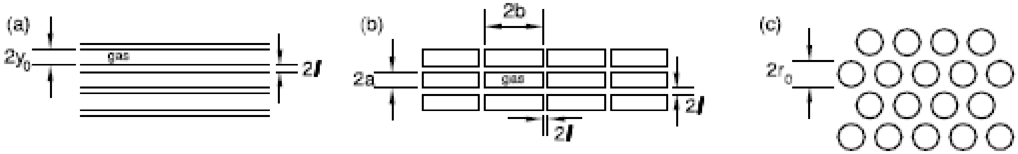
with f_v and f_κ the spatially averaged viscous and thermal function

Here: $f_j = \frac{tanh((1+i)y_0/\delta_j)}{(1+i)y_0/\delta_j}$ for $j = \kappa$ or ν

⇨ only known for common geometries



parallel-plates rectangular pores circular pores

Linear theory

From the governing equation, we can obtained :

- the thermoacoustic wave equation (in Fourier domain)[1]:

$$\rho_m \frac{d}{dx}\left(\frac{1-fv}{\rho_m}\frac{\partial p}{\partial x}\right) - \frac{1}{T_m}\frac{f_\kappa - fv}{1-\sigma}\frac{\partial Tm}{\partial x}\frac{\partial p}{\partial x} + \left(\frac{w}{c_0}\right)^2 [1+(\gamma-1)f_\kappa]p = 0$$

which can be written as two coupled first- order equations

$$dp = -\frac{iw\rho_m dx}{\phi S}\frac{1}{1-fv}u$$

$$du = -\frac{iw\phi S dx}{\gamma P_0}[1+(\gamma-1)f_\kappa]p + \boxed{\frac{f_\kappa - fv}{(1-f_v)(1-\sigma)}\frac{dT_m}{T_m}u}$$

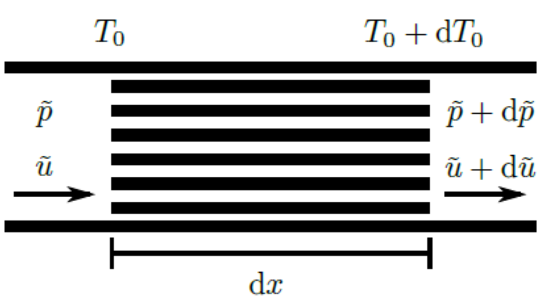
Lumped-element model of the regenerator

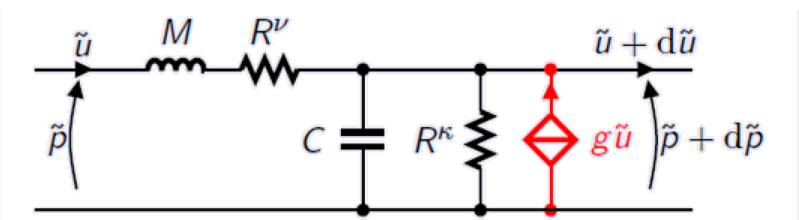
$$dp = (iwM + Rv)u$$

$$du = (iwC + 1/R^\kappa)p + gu$$

Electro-acoustic analogy

,





g ũ volumetric-velocity source with ***g*** amplification gain

[1] Rott, N. « Damped and thermally driven acoustic oscillations in wide and narrow tubes » , *ZAMP* 1969: 230-43.
[2] Gusev, V.; Bailliet, H.; Lotton, P.; Bruneau, M., Acta Acustica united with Acustica, Volume 86, Number 1, 2000, 25-38.

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- 2 quantities of interest in TA (second-order quantities):

    - Time-averaged acoustic works produced/absorbed per unit volume:

$$w_2 = \partial_x\left(\overline{p\langle v_x\rangle}\right)$$

    - Time-averaged thermoacoustic heat flux:

$$q_2 = \rho_m T_m \, \overline{s\langle v_x\rangle}$$

When the temperature distribution $T_m(x)$ is known, the acoustic quantities can be obtained

- Numerically,
- Analytically[2] (exact solution in the form of an infinite series of integral operator)

When the heat input Q_{hot} is imposed, a model of heat transfer in the core is required

[1] Rott, N. « Damped and thermally driven acoustic oscillations in wide and narrow tubes » , *ZAMP* 1969: 230-43.
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Stack with complex material and structure

In the classical linear theory: only one parameter (hydraulic radius) to describe the stack for standard geometry

Extension of this theory to porous material:

- tortuosity α_t introduced by Roh in 2007,

$$k_{eq} = \propto_t k_0[1 + (\gamma - 1) f_\kappa] \qquad dp = -\frac{iw\rho_m dx}{\phi S} \frac{1}{1 - f_v} \propto_t u$$

- Use of friction factor and heat-transfer coefficients [Swift and al.] which follow power laws in Reynolds number⇨ instantaneous value are determined from v(t) via steady flow correlation [kay]

- Johnson-Champoux-Allard model used by Dragonetti stack modelled as an "equivalent fluid" characterized by a complex density ρ and a complex bulk modulus K.

  fv and fk given by

$$f_v = 1 - \frac{1}{\alpha_\infty \left(1 + \frac{\phi\sigma}{j\omega\rho_m\alpha_\infty} \sqrt{1 + j\frac{4\alpha_\infty^2 \eta \rho_m \omega}{\phi^2\sigma^2\Lambda_v^2}}\right)}$$

$$f_k = 1 - \frac{1}{1 + \frac{8\eta}{j\Lambda_t^2 P_r \omega \rho_m} \sqrt{1 + j\frac{\Lambda_t^2 \rho_m P_r \omega}{16\eta}}}$$

The JCA model depends of five parameters : porosity ϕ , flow resistivity σ , tortuosity $\alpha\infty$, thermal characteristic length Λt and viscous characteristic length Λv

H. Roh, R. Raspet, H.E. Bass Parallel capillary-tube-based extension of thermoacoustic theory for random porous media, JASA., 121 (2007),
W. Swift and W. C. Ward. "Simple harmonic analysis of regenerators", Journal of Thermophysics and Heat Transfer, Vol. 10, No. 4 (1996),
Kays WM, London AL. Compact heat exchangers. New York: McGraw Hill;1964
Dragonetti, Raffaele, Marialuisa Napolitano, Sabato Di Filippo, et Rosario Romano. « Modeling energy conversion in a tortuous stack for thermoacostic applications ». Applied Thermal Engineering 103 (25 juin 2016): 233-42.

State of the art

Linear theory provides an analytical models based on 1D, steady-state operation

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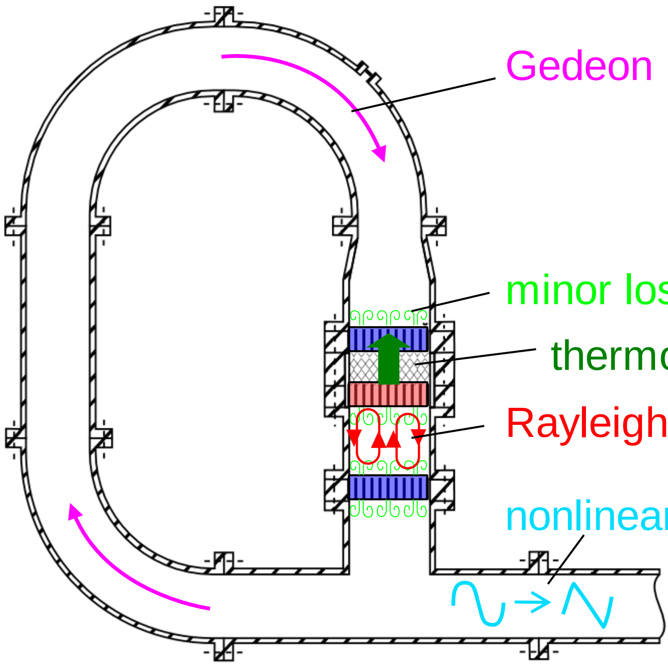
whereas in reality

- Onset of a self sustained acoustic wave controlled by linear effects

BUT saturation controlled by nonlinear effects ⇨ acoustic power dissipation or temperature/acoustic field modification



complicated processes, not fully described :

How to describe the nonlinear effects ?
How they impact the temperature field ?

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- Stack/regenerator with complex geometry, complicated heat transfer

mesh grids

NiCr foam

RVC foam

How to determine the viscous and thermal function ?
How to describe the temperature field for a given heat input ?
How the 3D temperature field impact the acoustic wave ?

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Physical acoustics or electro-acoustic

At LAUM, two main approaches ***: analytical and experimental***

New concept
+ compact design
+ active control of the TA amplification

Specific test benches for the characterization of the complex phenomena
+ measurement of transfer matrix of the thermoacoustic core
+ measurement of acoustic streaming with laser Doppler velocimetry
+ density fluctuations measurement with interferometric holography

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Thermoacoustics: an overview

I. What is thermoacoustics ?

II. Linear theory of thermoacoustic

III. Focus on

- Transfer matrix measurement
- Active tuning of acoustic oscillations

Transfer matrix measurement

Estimation of q_2 and w_2 required the knowledge of :

- the function $f\kappa$ and $f\upsilon$

mesh grids

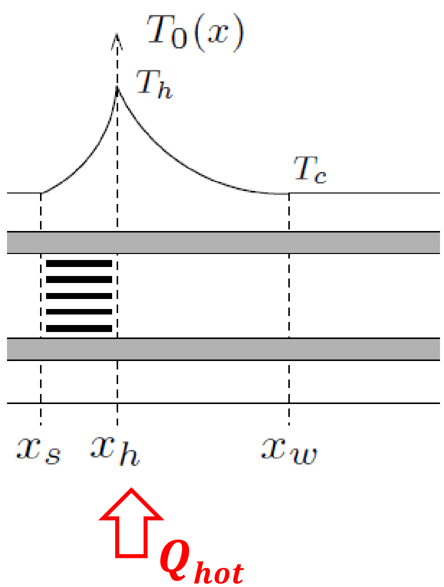
NiCr foam

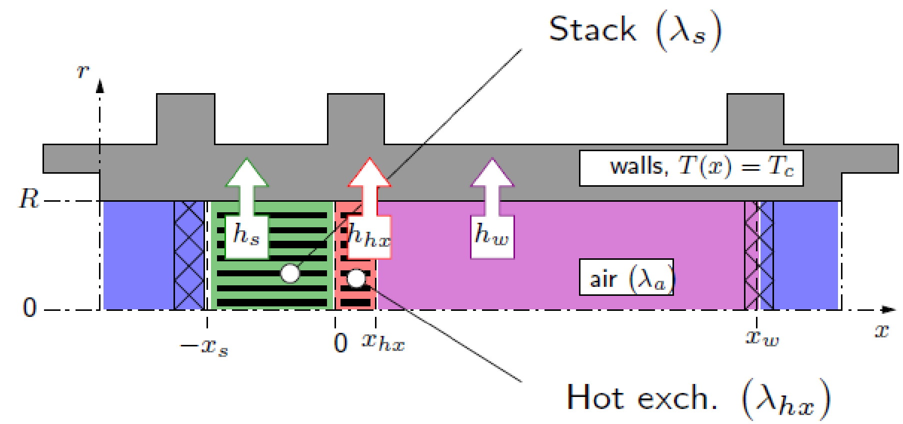
RVC foam

Transfer matrix measurement

Estimation of q_2 and w_2 required the knowledge of :

- the function $f\kappa$ and $f\nu$
- the thermophysical properties of the stack (need to know $T_o(x)$ from $\boldsymbol{Q_{hot}}$)



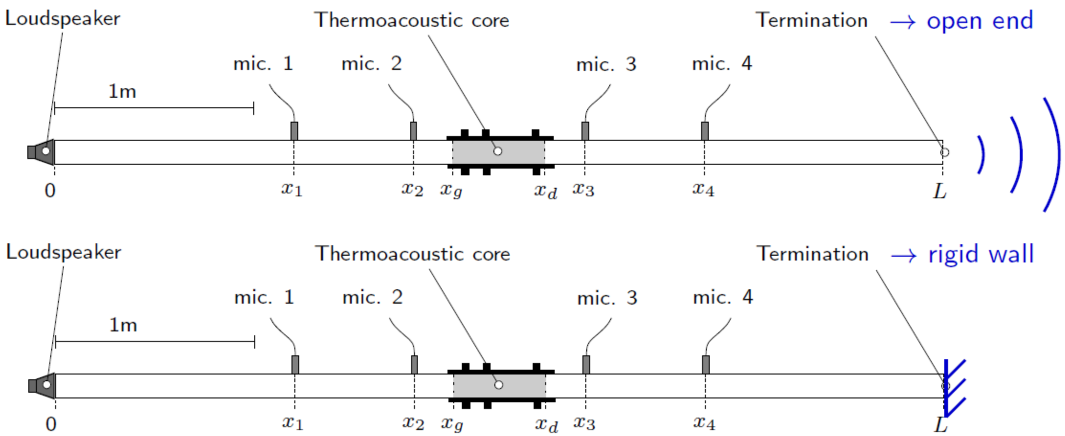


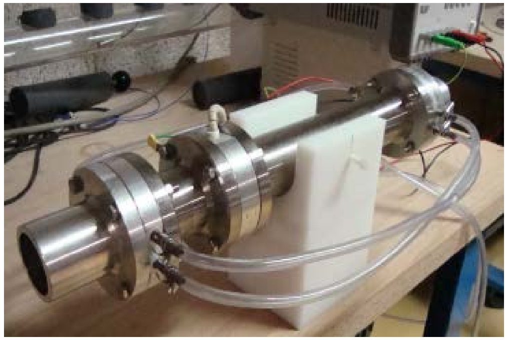
A solution : **« black box » approach**

⇨ Measure the transfer matrix of the TA core (under various heating conditions) to get around the difficulty to model heat transfer through its

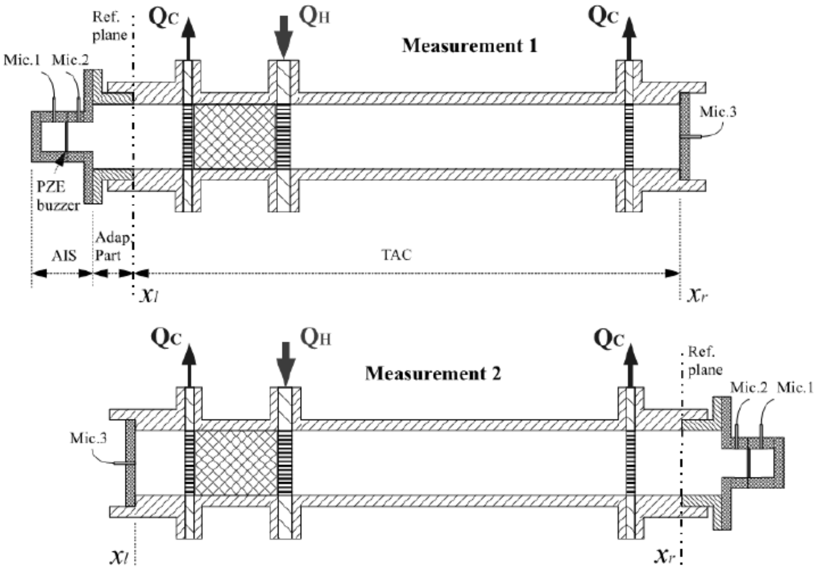
Transfer matrix measurement

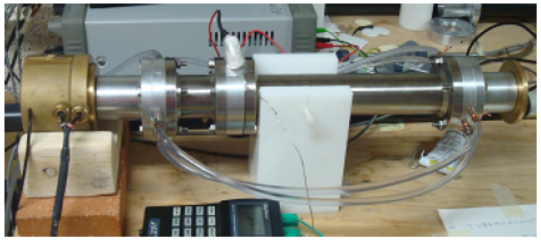
Using a two-load method

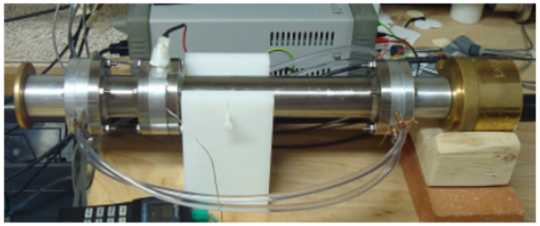




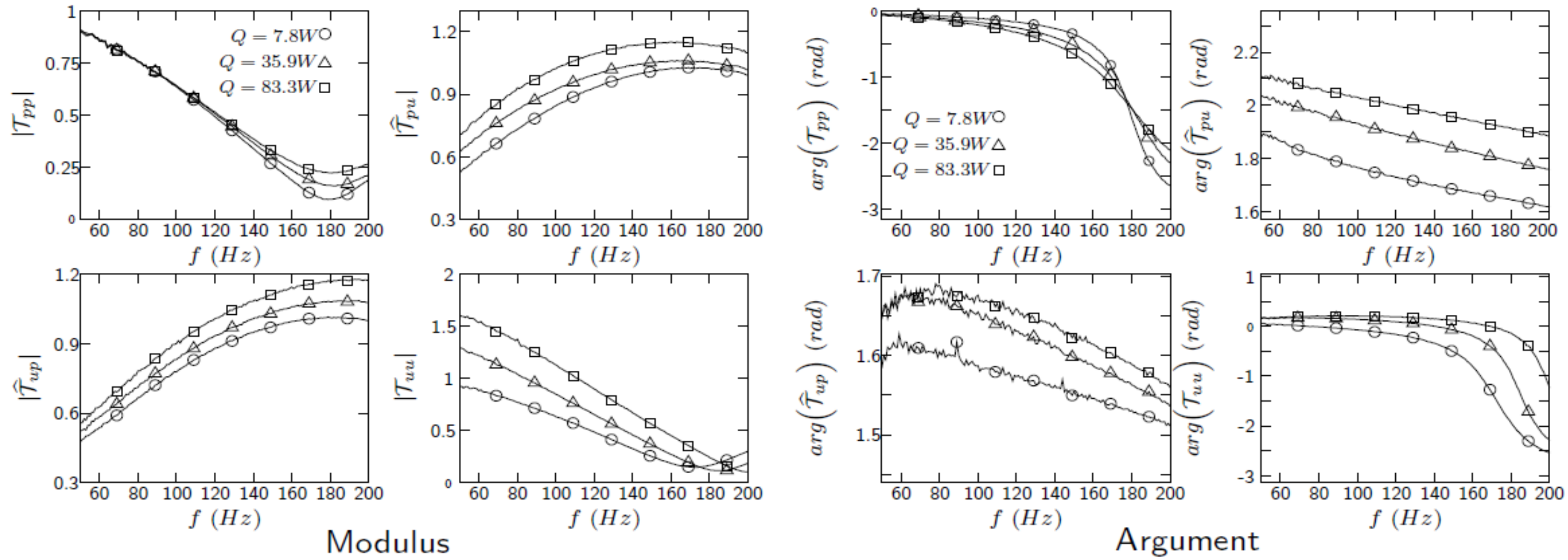
Using an acoustic impedance sensor







Transfer matrix measurement



From experimental T-matrix:

- Estimation of the onset of self-sustained oscillations [1]
- Comparation of the performances of different materials [2]
- Inverse problem to estimate the thermophysical properties of the stack [3]
- Optimal design of systems beforehand [2]

Prospect: Extension of the method to

- pressurized gas,
- higher amplitude to predict limit cycles [Zorgnotti *PhD Work In Progress*]

[1] M. Guédra et al., J. Acoust. Soc. Am., 2011
[2] F. Bannwart et al., J. Acoust. Soc. Am., 2013
[3] M. Guédra et al., Appl. Therm. Eng., 2014

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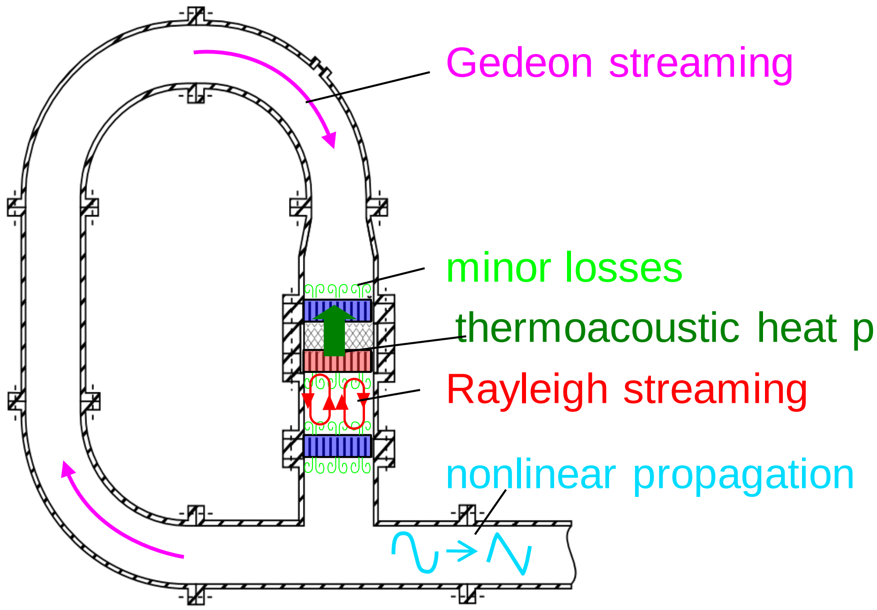
State of the art

Simple systems ... with a complex behaviour

Desgin tools used analytical models based on 1D linear theory, whereas in reality

- Onset of a self sustained acoustic wave (at the frequency of the most unstable mode) controlled by linear effects but saturation controlled by nonlinear effects acoustic power dissipation or temperature/acoustic field modification



How to describe the nonlinear effects ?
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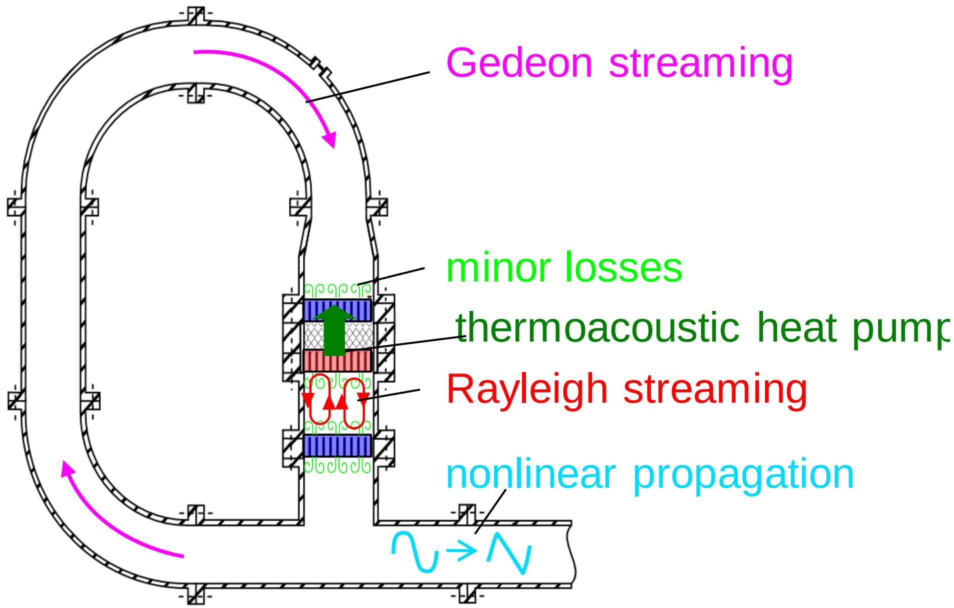
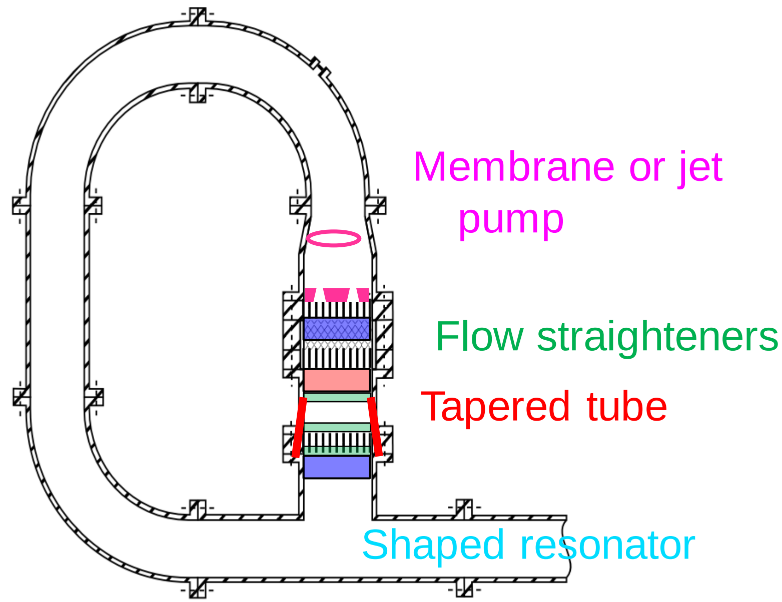
How to control the acoustic field distribution ?

Common solution:
Add passive elements (semi-empirically designed)





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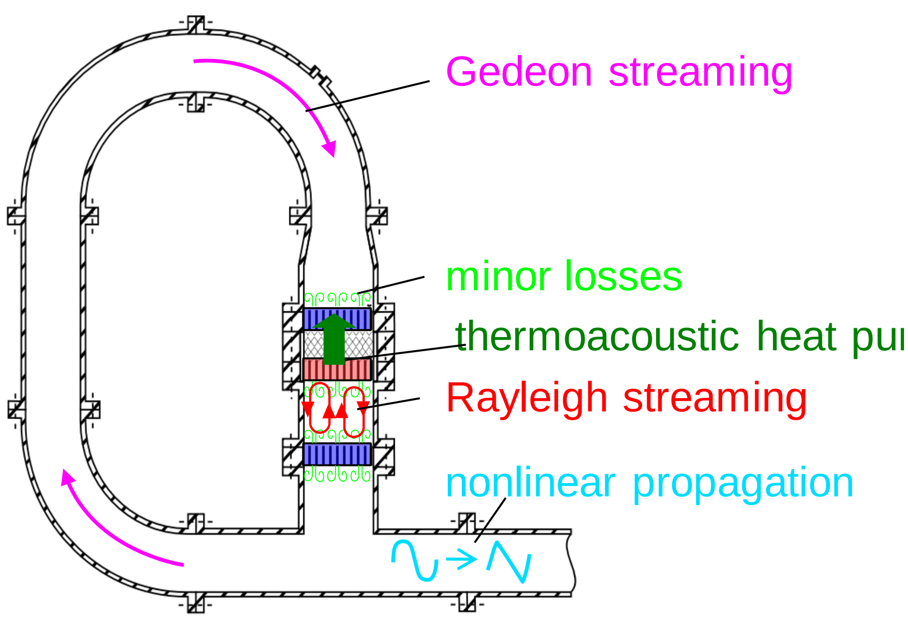
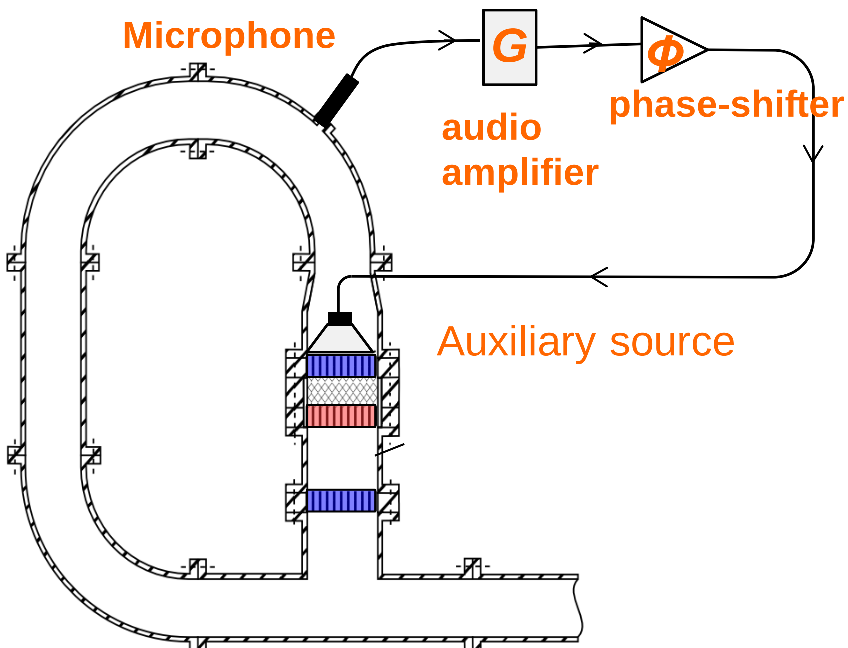
How to control the acoustic field distribution ?

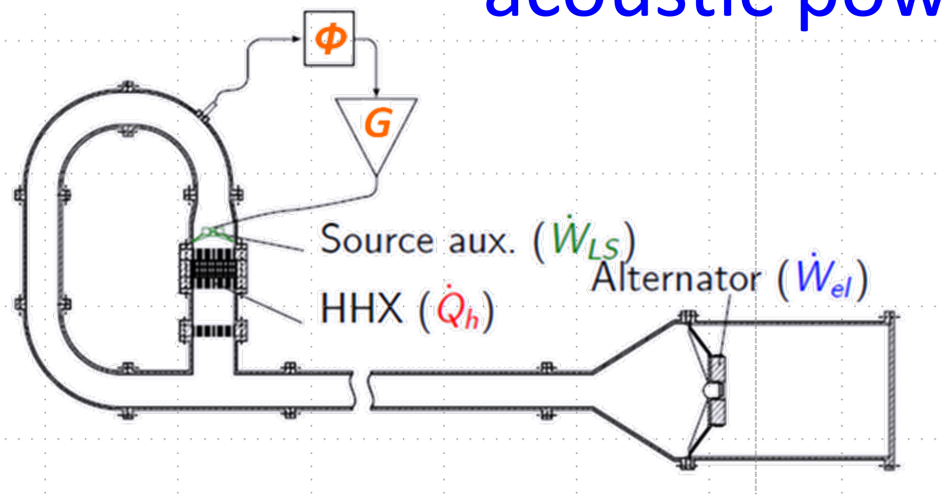
New approach : active control method → Add an electro-acoustic feedback loop to external force the self sustained wave





[1] C. Olivier, G. Penelet, G. Poignand and P. Lotton . « Active control of thermoacoustic amplification in a thermo-acousto-electric engine », Journal of Applied Physics, vol. 115 [17], 2014.

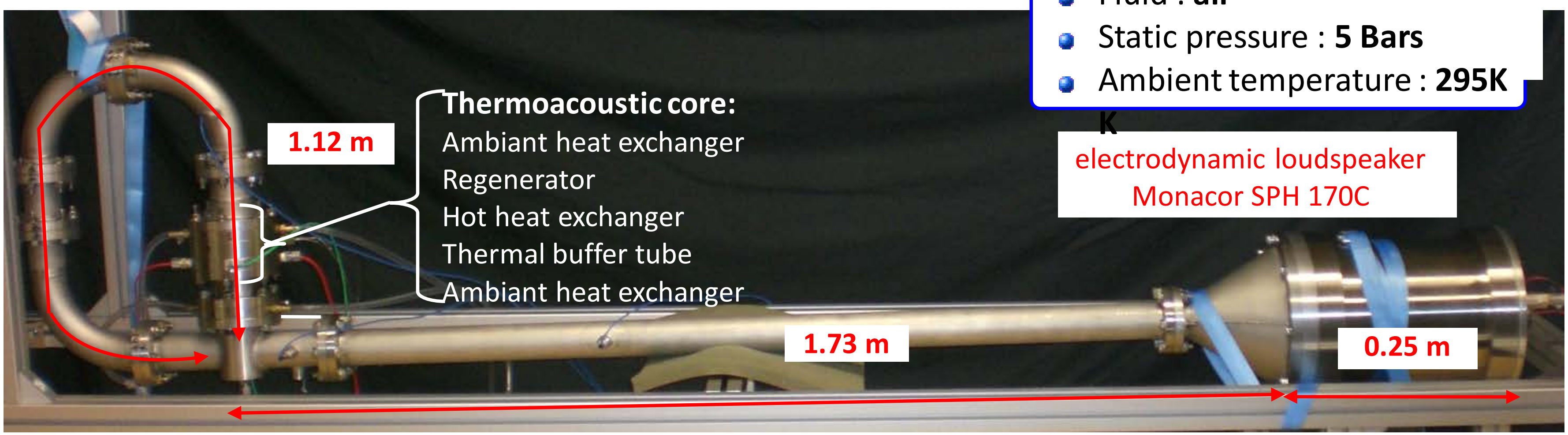
Active tuning of acoustic oscillations in a thermo-acoustic power generator



without feedback loop $$\eta_\phi = \frac{\dot{W}_{el}}{\dot{Q}_h}$$

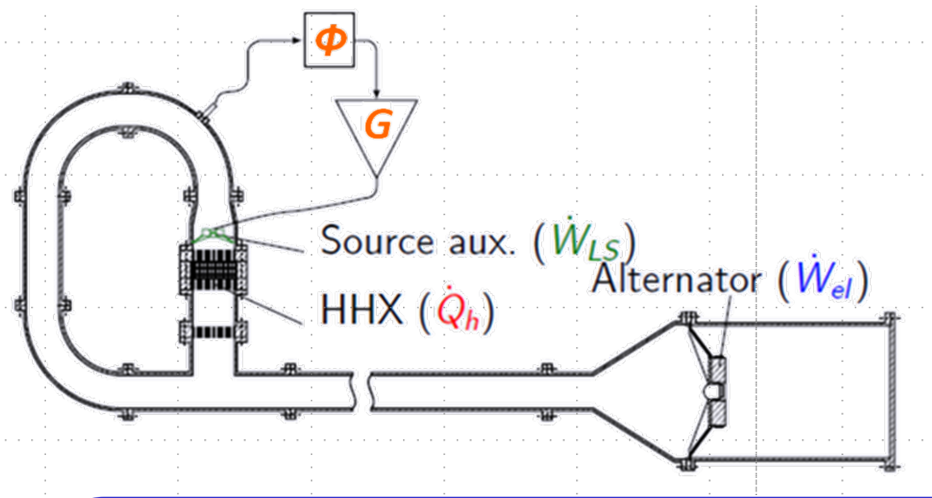
with feedback loop $$\eta = \frac{\dot{W}_{el}}{\dot{Q}_h + \dot{W}_{LS}}$$

- Fluid : **air**
- Static pressure : **5 Bars**
- Ambient temperature : **295K**
  K



- Frequency: **40 Hz**
- $\boldsymbol{\eta_{max}}$ **= 1 %,** $\boldsymbol{Pel_{max}}$ **= 1W**
- Low efficiency: engine = study model (modular, limited budget, low efficiency alternator) but designed to work closed to its maximum value.

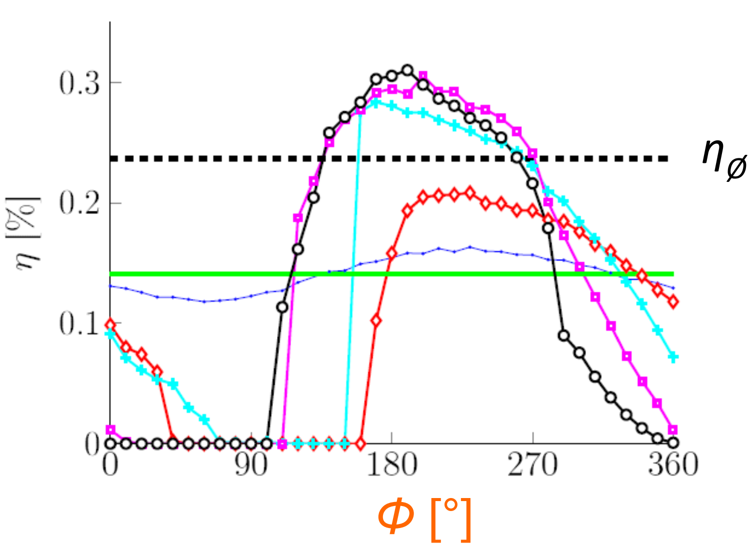
Efficiency *η* versus *φ* for different *G*

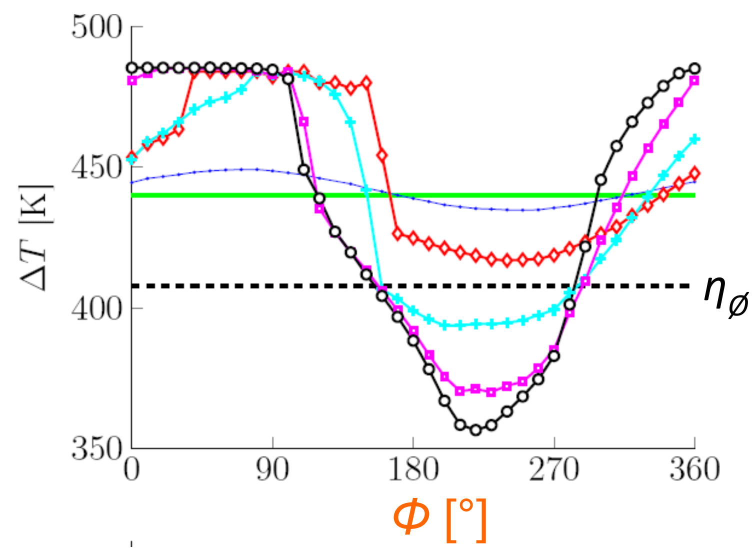


without feedback loop $\eta_\phi = \dfrac{\dot{W}_{el}}{\dot{Q}_h}$

with feedback loop $\eta = \dfrac{\dot{W}_{el}}{\dot{Q}_h + \dot{W}_{LS}}$

Q_h = 70 W, *G* = 0 (-), 10 (-), 40 (◊), 70 (+) , 135 (□) or 190 (o)





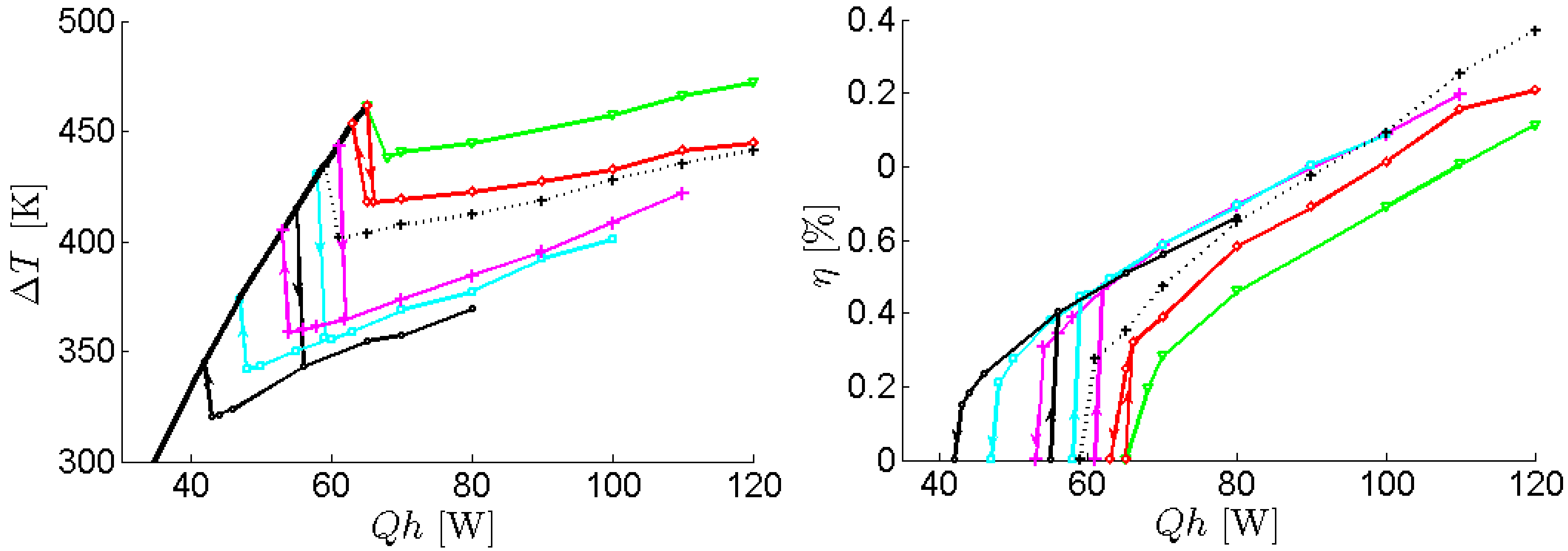
- *Efficiency improvement:* efficiency η higher than the one without active control $\eta_\varnothing$

Hysteresis behaviour

Method : 1. Search onset condition, $Q_h \nearrow$
2. Above onset : Efficiency measurement when $Q_h \nearrow$ and then $Q_h \searrow$
3. Search offset condition

Steady-state measurements

φ = φ$_{opt}$, *G* = 0 (-), 40 (◊), 70 (+) , 135 (□) or 190(o)



- hysteresis behaviour: offset temperature ΔT_{offset} lower than onset temperature ΔT_{onset}
- With the gain *G*, $\Delta T_{\text{offset}} \searrow$, $\Delta T_{\text{onset}} < \Delta T_{\varnothing\ onset}$, $\eta > \eta_{\varnothing}$

Conclusions

Active control works, but why ?

- Need of a simplified model to get better comprehension

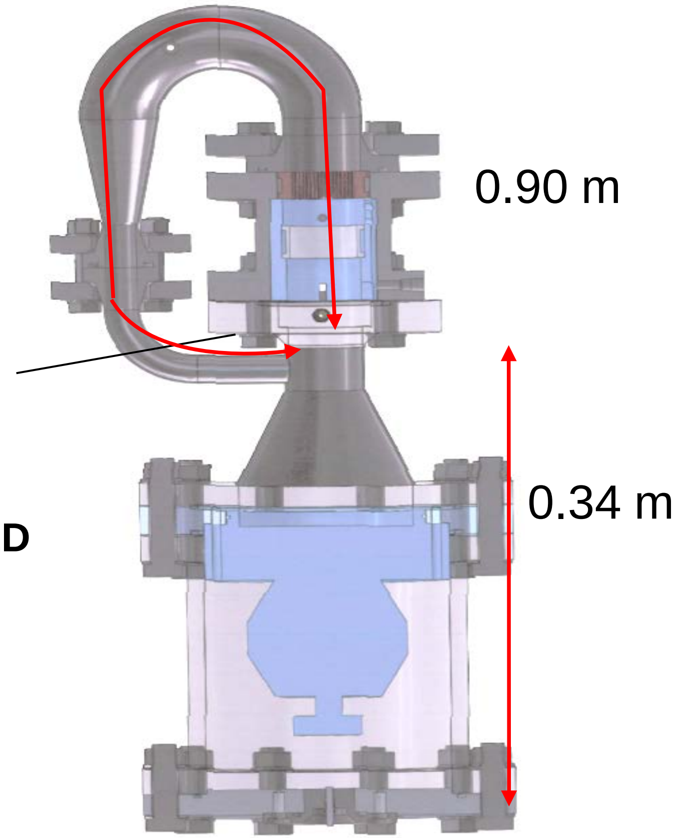
\+ Application to a higher power TA engine

Under construction

- Fluid : **helium**
- Static pressure : **22 Bars**

- Heat input : **1000 W**
- Efficiency (theoretical): **20 %**
- Electric power: **200 W**

alternator:
Qdrive 1S 132D



Thank you !

Gaëlle POIGNAND
Laboratoire d'Acoustique de l'Université du Mans
UMR CNRS 6613

Which simulation tools used to describe the behaviour of the system ?

- Qualitative description (capture all the most important features, basic insights into the operation of the engine)
- Both transient and steady-state operation

Tools available

- Design software tools

  BUT linear approx

  steady states

- Direct Numerical Simulation

  quantitative description

  BUT large computation times due to complicated physics and multiple time and space scales

Here simple model

1. Description of the acoustic propagation

   Approach based on nonlinear dynamics: ordinary differential equations given by lumped element
2. Description of the temperature distribution

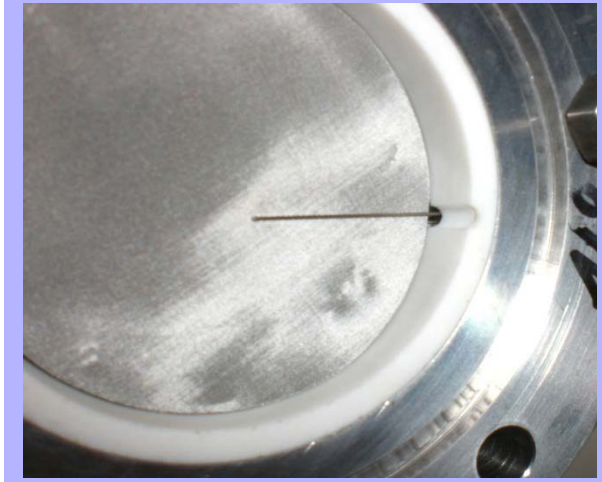
   Resolution of unsteady heat transfer through the thermoacoustic core based on a finite difference scheme

Travelling wave thermoacoustic engine part

Cold heat exchanger

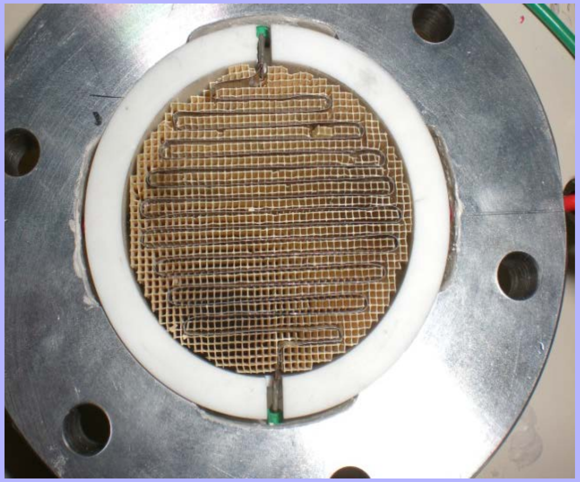
Copper block with 2 mm diameter drilled holes.
Water circulates around the block
Porosity: 69 %
Length: 1.5 cm

Regenerator



Stainless steel wire mesh
Porosity: 69 %
Hydraulic radius: 20 µm
Length: 2.3 cm

Hot heat exchanger



Ceramic stack with two ribbon heaters
Length: 1.5 cm
Qh max = 235 W
(Rribbon = 4.7 Ω)

Inverse method for the estimation of acoustical and thermal parameters

Estimation of acoustical parameters

Other Materials

Wire Mesh | NiCr Foam | RVC Foam

[F. Bannwart *et al.* in *Acoustics 2012* Conf. Proc.]

	wire mesh	NiCr foam	RVC foam
ϕ	0.68	0.91	0.97
r_s	$40\ \mu m$	$0.31\ mm$	$0.17\ mm$
α_t	1.06	1.29	1.13

Table 1: inverse method results

	wire mesh	NiCr foam	RVC foam
ϕ	0.46	–	0.97
r_s	$45\ \mu m$	$0.3\ mm$	$0.13\ mm$
α_t	–	–	–

Table 2: manufacturer data

1. Estimation is made for 3 parameters :
   - porosity ϕ_s,
   - channel's inner radius r_s,
   - tortuosity α_t.
2. Properties of hot exchanger are those previously estimated.

RVC Foam

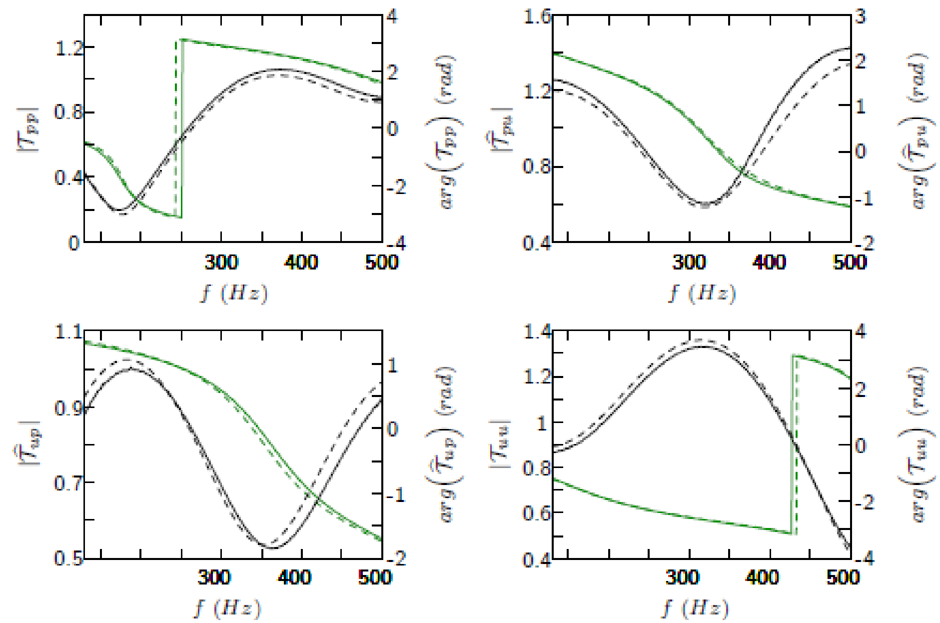


Figure 10: Theoretical transfer matrix fitted on experiments.

Stack ≡ tortuous porous material

Johnson-Champoux-Allard model: stack modelled as an "equivalent fluid" characterized by a complex density ρ and a complex bulk modulus K.

fv and fk given by

$$f_v = 1 - \frac{1}{\alpha_\infty \left(1 + \frac{\phi\sigma}{j\omega\rho_m\alpha_\infty}\sqrt{1 + j\frac{4\alpha_\infty^2\eta\rho_m\omega}{\phi^2\sigma^2\Lambda_v^2}}\right)}$$

$$f_k = 1 - \frac{1}{1 + \frac{8\eta}{j\Lambda_t^2 P_r\omega\rho_m}\sqrt{1 + j\frac{\Lambda_t^2\rho_m P_r\omega}{16\eta}}}$$

The JCA model depends of five parameters : porosity ϕ , flow resistivity σ , tortuosity $\alpha\infty$, thermal characteristic length Λt and viscous characteristic length Λv

≠ only one parameter (hydraulic radius) to describe the standard stack with straight pores.

Dragonetti, Raffaele, Marialuisa Napolitano, Sabato Di Filippo, et Rosario Romano. « Modeling energy conversion in a tortuous stack for thermoacostic applications ». Applied Thermal Engineering 103 (25 juin 2016): 233-42.

Napolitano, Marialuisa, Raffaele Dragonetti, et Rosario Romano. « A method to optimize the regenerator parameters of a thermoacoustic engine ». Energy Procedia, ATI 2017 - 72nd Conference of the Italian Thermal Machines Engineering Association, 126 (sep 2017): 525-32.

TA quantities of interest

Acoustic works produced/absorbed per unit volume: $\langle w_2 \rangle = \partial_x\left(\overline{p\langle v_x \rangle}\right)$

$$\langle w_2 \rangle = \langle w_\kappa \rangle + \langle w_\nu \rangle + \langle w_{SW} \rangle + \langle w_{TW} \rangle$$

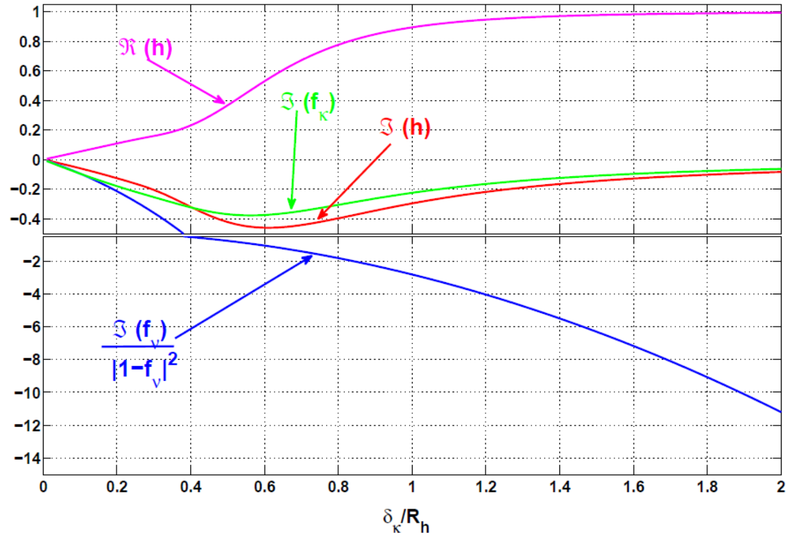
with
$$\langle \mathfrak{w}_\kappa \rangle = \frac{1}{2}\frac{\gamma - 1}{\rho_m c_0^2}\boxed{\Im(f_\kappa)}\omega\, |\tilde{p}|^2$$

$$\langle \mathfrak{w}_\nu \rangle = \frac{1}{2}\omega\rho_m \boxed{\frac{\Im(f_\nu)}{|1 - f_\nu|^2}}\, |\langle \tilde{v} \rangle|^2$$

$$\langle \mathfrak{w}_{SW} \rangle = -\tfrac{1}{2}\Im\left(\tilde{p}\langle \tilde{v}_x^* \rangle\right)\, \frac{d_x T_m}{T_m}\boxed{\Im(h)}$$

$$\langle \mathfrak{w}_{TW} \rangle = \tfrac{1}{2}\Re\left(\tilde{p}\langle \tilde{v}_x^* \rangle\right)\, \frac{d_x T_m}{T_m}\boxed{\Re(h)}$$

$$h = \frac{f_\kappa - f_\nu}{(1 - \sigma)(1 - f_\nu)}$$



Tominaga, A. « Thermodynamic aspects of thermoacoustic theory ». Cryogenics 35, nº 7, 1995.

TA quantities of interest

thermoacoustic heat flux : $q_2 = \rho_m T_m \,\overline{s\langle v_x\rangle}$

$$\langle q_2\rangle = \langle q_{SW}\rangle + \langle q_{TW}\rangle + \lambda_{ac}\partial_x T$$

with

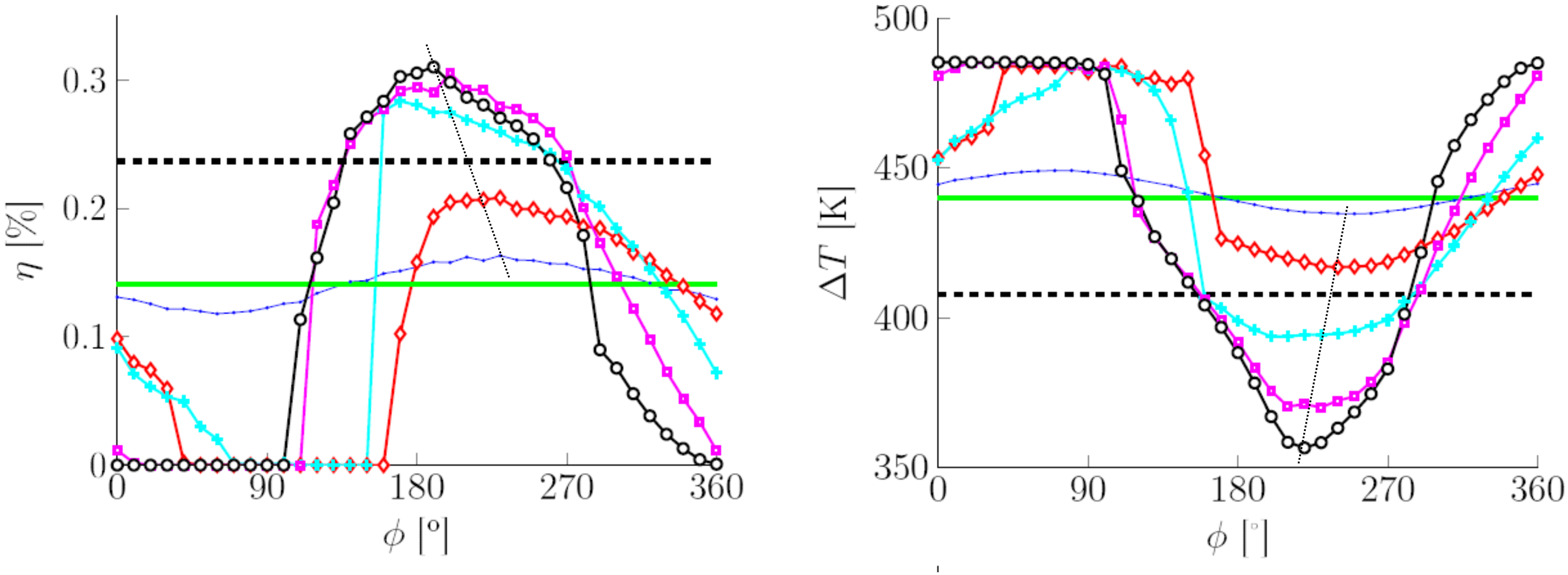
$$\langle q_{SW}\rangle = -\frac{1}{2}\Im\left(\tilde{p}\langle\tilde{v}_x^*\rangle\right)\Im(g) \qquad g = \frac{f_\nu^* - f_\kappa}{(1+\sigma)\,(1-f_\nu^*)}$$

$$\langle q_{TW}\rangle = \frac{1}{2}\Re\left(\tilde{p}\langle\tilde{v}_x^*\rangle\right)\Re(g)$$

$$\langle q_D\rangle = -\frac{\rho_m C_p}{2\omega(1-\sigma^2)} d_x T_m \Im(g_D)\left|\langle\tilde{v}\rangle\right|^2 \qquad g_D = \frac{(\sigma f_\nu^* - f_\kappa)}{\left|1-f_\nu\right|^2}$$

Efficiency *η* versus *φ* for different *G*

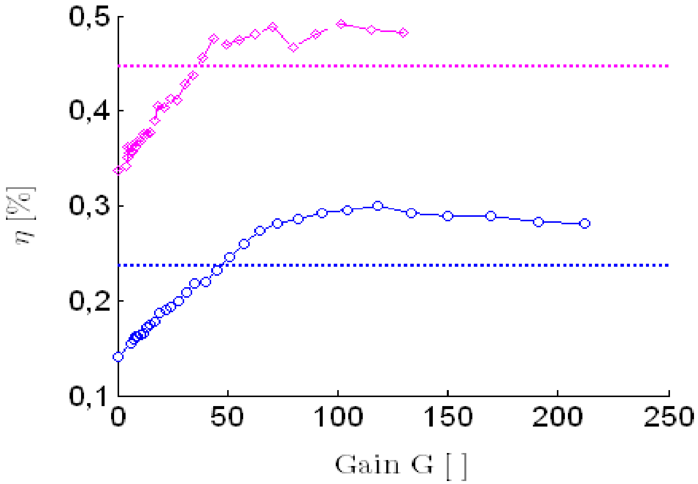
Q_h = 70 W, *G* = 0 (-), 10 (-), 40 (◊), 70 (+) , 135 (□) or 190(o), without active control (--)

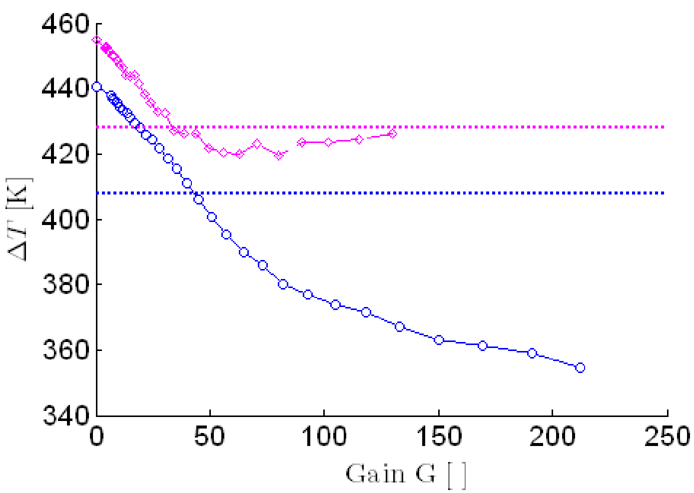


- with the phase φ : - *η* varies ⇨ optimal phase φ_{opt} (varies with the gain)
  - acoustic wave death
- when the gain *G* ↗: - *η* ↗ and ΔT ↘
- for high *G* : - $\eta > \eta_{\varnothing}$, $\Delta T > \Delta T_{\varnothing}$ ⇨ nonlinear interaction ?

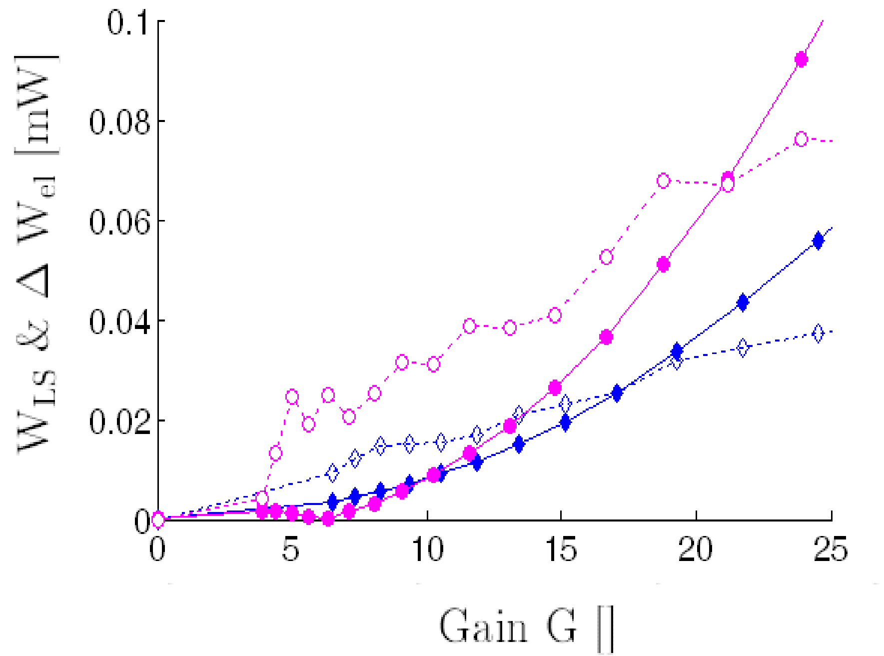
W_{ls} and W_{el} versus G for $\varphi = \varphi_{opt}$

Q_h = 70 W (o), 100W (◇), without active control (..)





ΔW_{el} (◇,o) additional power produced
W_{ls} (◆,•) power supllied to AC source



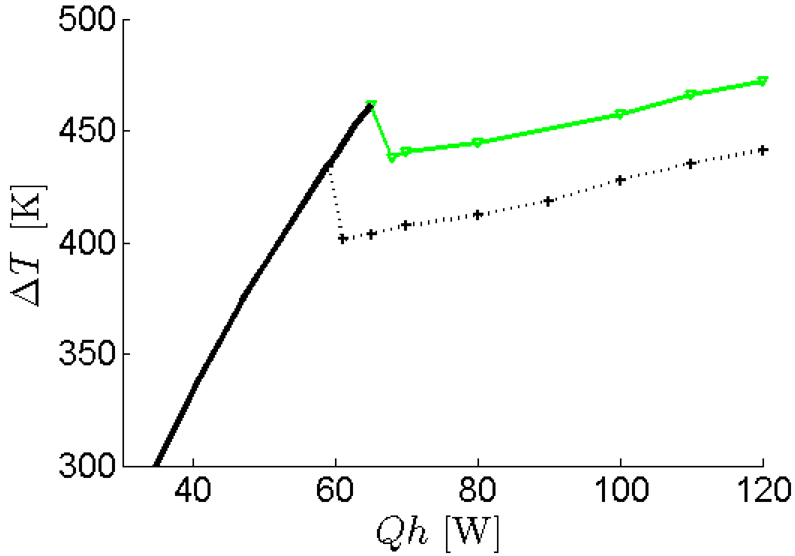
- efficiency improvement saturates
- configurations for which $\Delta Wel > W_{LS}$

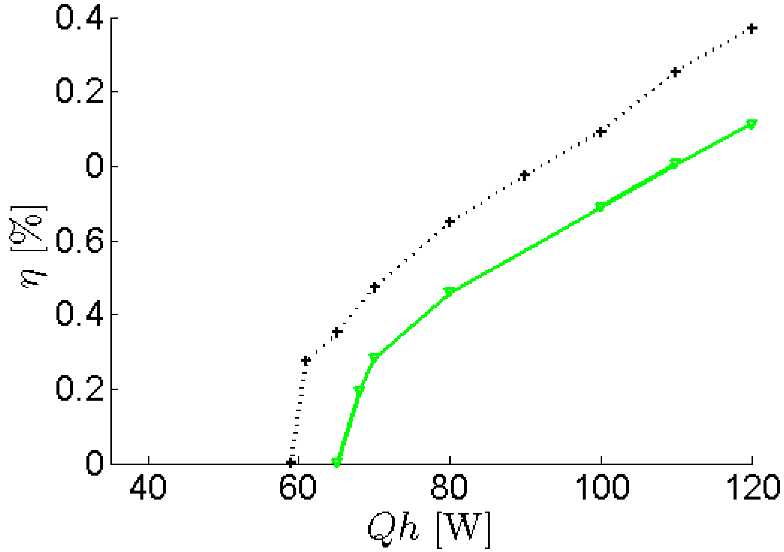
NB: $\eta = \dfrac{Wel\,(G=0)+\Delta Wel}{Q_{h}+W_{ls}}$

Hysteresis behaviour

Method : 1. Search onset condition, Qh ↗
2. Above onset : Efficiency measurement when Qh ↗ and then Qh ↘
3. Search offset condition
Steady-state measurements

$\varphi = \varphi_{opt}$, $G = 0$ (-)
without active control (..)

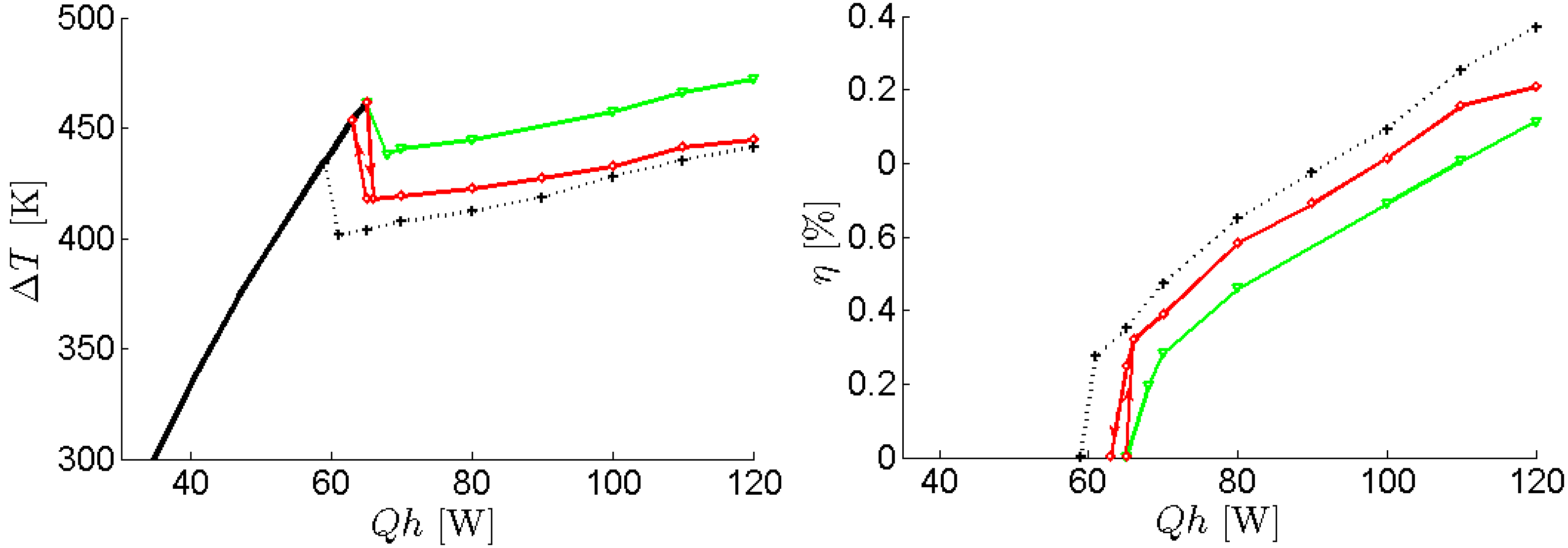




- For $G = 0$, $\Delta T_{onset} > \Delta T_{\varnothing\ onset}$ and $\eta < \eta_{\varnothing}$

Hysteresis behaviour

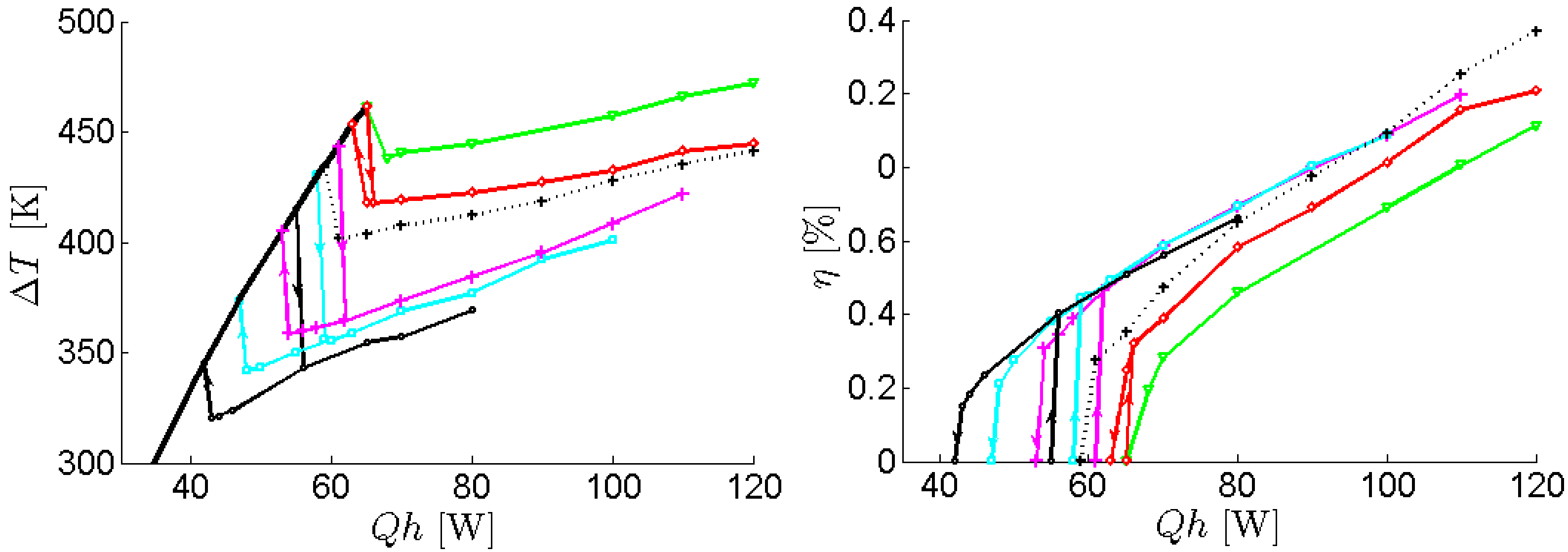
$\varphi = \varphi_{opt}$, G = 0 (-), 40 (◊)
without active control (..)



- For G = 0, $\Delta T_{\text{onset}} > \Delta T_{\text{Ø onset}}$ and $\eta < \eta_{Ø}$
- For $G \neq 0$, hysteresis behaviour: $\Delta T_{offset} < \Delta T_{onset}$, system works for $Qh < Qh_{onset}$

Hysteresis behaviour

φ = φ_{opt}, *G* = 0 (-), 40 (◊), 70 (+) , 135 (□) or 190(**o**),
without active control (..)

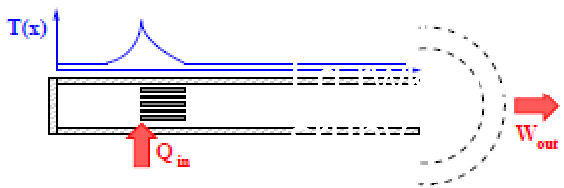


- For *G* = 0, $\Delta T_{\text{onset}} > \Delta T_{\varnothing\ onset}$ and $\eta < \eta_{\varnothing}$
- For *G* ≠ 0, hysteresis behaviour: $\Delta T_{offset} < \Delta T_{onset}$, system works for $Qh < Qh_{onset}$
- With the gain *G*, ΔT_{offset}↘, $\Delta T_{\text{onset}} < \Delta T_{\varnothing\ onset}$, $\eta > \eta_{\varnothing}$

From G. Penelet n3L Summer school, Munich 2013

Autonomous thermoacoustic oscillators : two examples

TA Laser

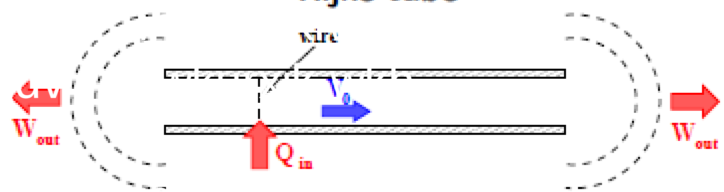


$$\dot{q} = f(d_x T, p, v_x, stack \ldots)$$

⋆ not acoustically compact

⋆ Paradigmatic of TA engines
(energetic applications)

Rijke tube



$$\dot{q} = f(T_{wire}, v_x, V_0 \ldots)$$

⋆ acoustically compact
$(\dot{q} \approx \dot{q}(t)\delta(x - x_{wire}))$

⋆ Paradigmatic of combustion instabilities

Common features

⋆ Autonomous oscillators driven by heat (which both fill the "Rayleigh criterium" $\int p\dot{q}.dt > 0$)
⋆ Simple in terms of geometry, but complicated operation above threshold . . .

Simple harmonic analysis of regenerators

G. W. Swift; W. C. Ward

The flow at any instant of time has no memory of its recent history -> viscous drag and heat transfer are determined only by the instantaneous velocity, with no dependance on the velocity at earlier times

Strong assumption : gas displacement should be larger than a pore dimension)

$$\frac{dp_1}{dx} = -i\omega\rho_m\left[1+\frac{(1-\phi)^2}{2(2\phi-1)}\right]\langle u_1\rangle - \frac{\mu}{r_h^2}\left(\frac{c_1(\phi)}{8}+\frac{c_2(\phi)N_{R,1}}{3\pi}\right)\langle u_1\rangle\,, \qquad (10.7$$

$$\begin{aligned}\frac{d\langle u_1\rangle}{dx} &= -\frac{i\omega\gamma}{\rho_m a^2}p_1 + \beta\frac{dT_m}{dx}\langle u_1\rangle + \\ & i\omega\beta\left[\frac{T_m\beta}{\rho_m c_p}\frac{\epsilon_s+(g_c+e^{2i\theta_p}g_v)\epsilon_h}{1+\epsilon_s+(g_c+e^{2i\theta_T}g_v)\epsilon_h}p_1 - \frac{1}{i\omega}\frac{dT_m}{dx}\frac{\epsilon_s+(g_c-g_v)\epsilon_h}{1+\epsilon_s+(g_c+e^{2i\theta_T}g_v)\epsilon_h}\langle u_1\rangle\right]\end{aligned}$$

$$c_1(\phi) = 1268 - 3545\phi + 2544\phi^2, \quad c_2(\phi) = -2.82 + 10.7\phi - 8.6\phi^2,$$

$$b(\phi) = 3.81 - 11.29\phi + 9.47\phi^2,$$

$$N_{R,1} = 4\left|\langle u_1\rangle\right| r_h\rho_m/\mu,$$

$$\epsilon_s = \phi\rho_m c_p/(1-\phi)\rho_s c_s, \quad \epsilon_h = 8ir_h^2/b(\phi)\sigma^{1/3}\delta_\kappa^2,$$

$$\delta_\kappa^2 = 2k/\omega\rho_m c_p,$$

$$\theta_p = \text{phase}(\langle u_1\rangle) - \text{phase}\,(p_1)\,, \quad \theta_T = \text{phase}(\langle u_1\rangle) - \text{phase}\left(\langle T\rangle_{u,1}\right),$$

$$g_c = \frac{2}{\pi}\int_0^{\pi/2}\frac{dz}{1+N_{R,1}^{3/5}\cos^{3/5}(z)}, \quad g_v = -\frac{2}{\pi}\int_0^{\pi/2}\frac{\cos(2z)\,dz}{1+N_{R,1}^{3/5}\cos^{3/5}(z)}.$$