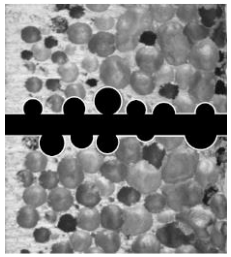
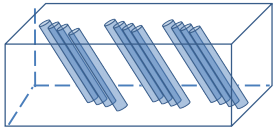
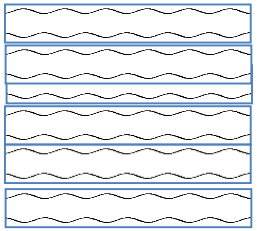
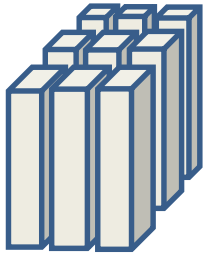
SAPEM
Symposium on the Acoustics of Poro-Elastic Materials
Le Mans, France, December 6-8, 2017

The Open University









Microstructures for lowering the quarter wavelength resonance frequency of a hard-backed rigid-porous layer

Keith Attenborough
Engineering and Innovation
The Open University, Milton Keynes, UK

*Work stimulated by a **3D-printing project proposal** for EU CA15125 DENORMS.*
Greater detail can be found in Applied Acoustics Vol 130
pp 188–194 January 2018

- For useful low frequency resonant absorption, what can be achieved using simple microstructures?

Identical parallel uniform pores

[M R Stinson, J. Acoust. Soc. Am. 1991; 89: 550 -8]

single pore

complex density $\rho(\omega) = \rho_0 / H(\lambda)$

complex compressibility $C(\omega) = (\gamma P_0)^{-1}[\gamma - (\gamma - 1)H(\lambda\sqrt{(N_{PR})})]$

Pore Shape	λ	$H(\lambda)$
slit (width $2b$)	$b\sqrt{(\omega/\nu)}$	$1 - \tanh(\lambda\sqrt{-i})/(\lambda\sqrt{-i})$
cylinder (radius a)	$a\sqrt{(\omega/\nu)}$	$1 - (2/\lambda\sqrt{i})J_1(\lambda\sqrt{i})/J_0(\lambda\sqrt{i})$
equilateral triangle (side d)	$(d\sqrt{3}/4)\sqrt{(\omega/\nu)}$	$1 - 3\coth(\lambda\sqrt{-i})/(\lambda\sqrt{-i}) + 3i/\lambda^2$
rectangle (sides $2a$,$2b$)	$\frac{2ab}{\pi\sqrt{(a^2+b^2)}}\sqrt{(\omega/\nu)}$	$\frac{-4i\omega}{\mu a^2 b^2}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left\{\alpha_k^2\beta_l^2\left[\alpha_k^2+\beta_l^2-\left(\frac{i\omega}{\mu}\right)\right]^{-1}\right\}$, $\alpha_k = \left(k+\frac{1}{2}\right)\left(\frac{\pi}{a}\right)$, $\beta_l = \left(l+\frac{1}{2}\right)\left(\frac{\pi}{b}\right)$

bulk material

porosity

tortuosity *angle to surface normal*

$$\rho_b(\omega) = (T(\theta)/\Omega)\rho(\omega), \quad C_b(\omega) = \Omega C(\omega) \qquad T(\theta) = 1/\cos^2(\theta)$$

$$k(\omega) = \omega\sqrt{[\rho_b(\omega)C_b(\omega)]} \qquad Z_c(\omega) = (\rho_0 c_0^2)^{-1}\sqrt{[\rho_b(\omega)/C_b(\omega)]}$$

hard-backed layer

Surface impedance

$$Z(d) = Z_c\coth(-ikd)$$

Normal incidence absorption coefficient

$$\alpha(d) = 1 - \left|\left(R(d)\right)^2\right|$$

Plane wave reflection coefficient

$$R(d) = \frac{\rho_0 c_0 - Z(d)}{\rho_0 c_0 + Z(d)}$$

flow resistivity

$$R_s = \frac{2\mu T s_0}{\Omega r_h^2}$$

shape factor

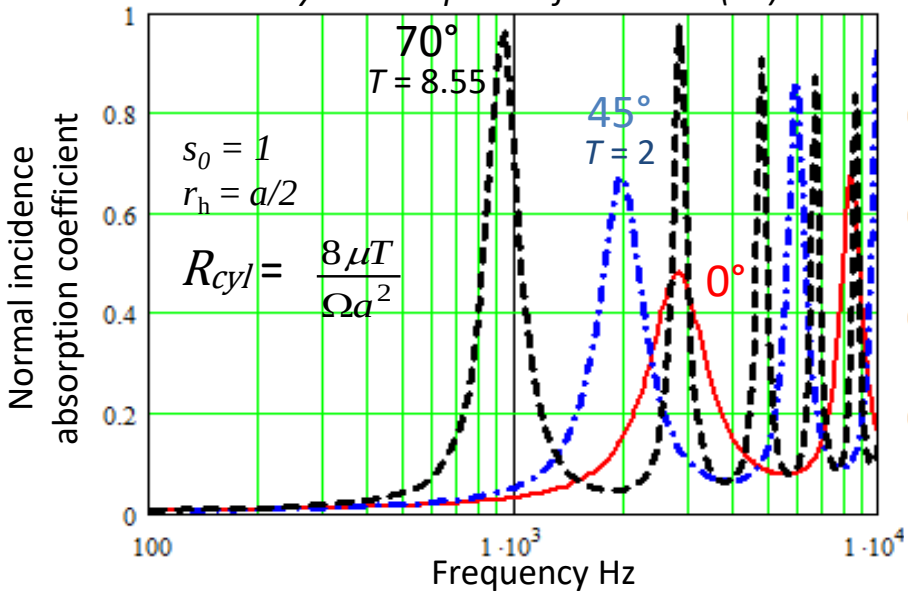
hydraulic radius

[Koseny – Carman]

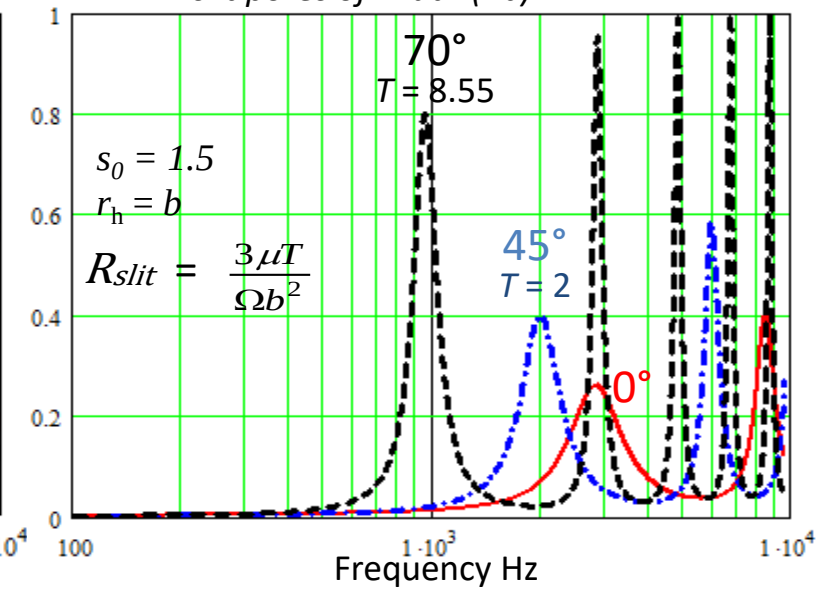
Pore shape and inclination

0.03 m hard-backed thick layer, porosity 0.3

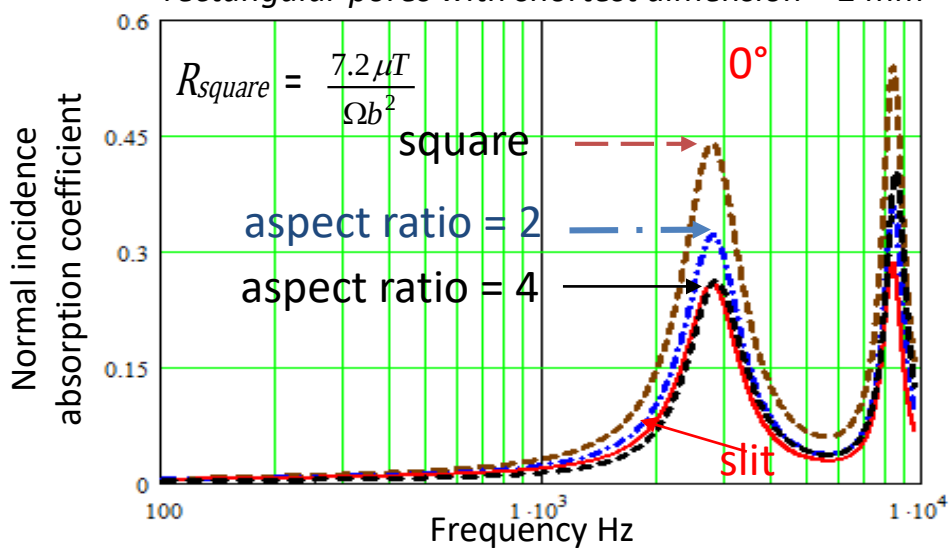
cylindrical pores of diameter (2a) = 2 mm



slit pores of width (2b) = 2 mm



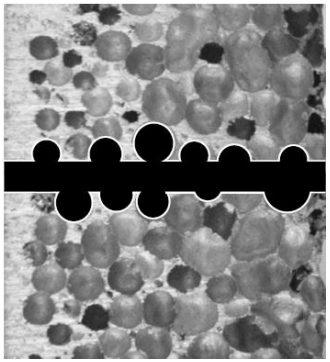
rectangular pores with shortest dimension = 2 mm

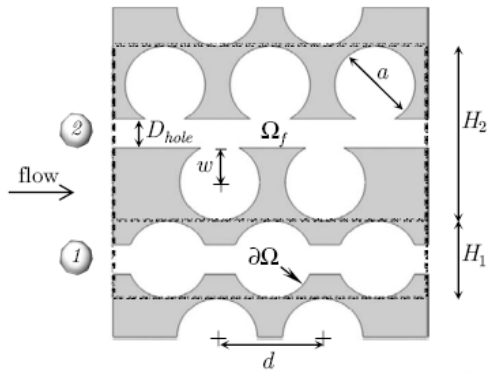


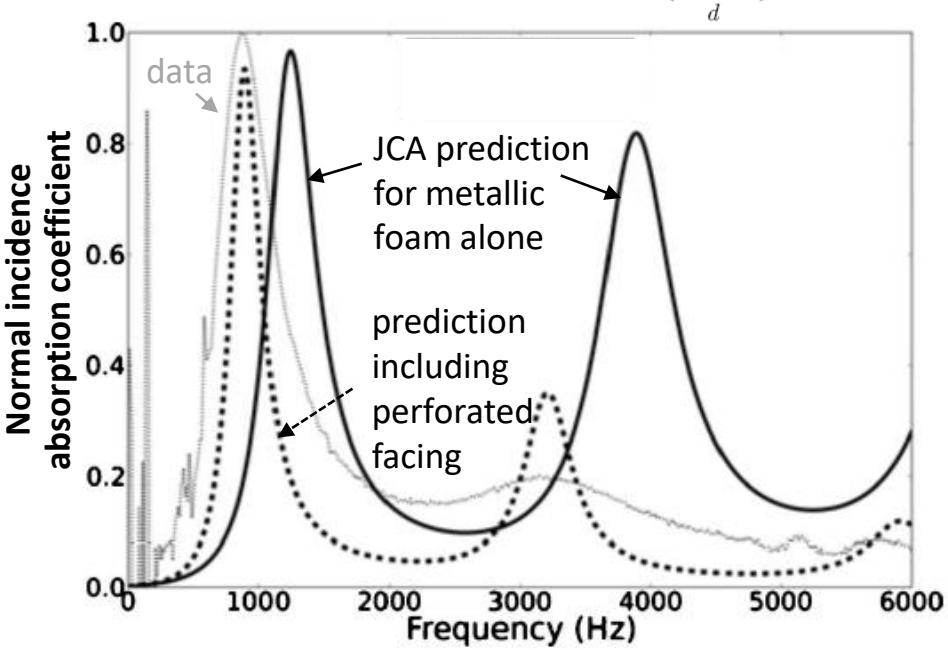
0.0264 m thick perforated metallic foam

from Chevillotte *et al* JASA 2010

estimated JCA parameters porosity 0.283, flow resistivity 8.4 kPa s m^{-2}, tortuosity 5.54, viscous characteristic length 0.281 mm, thermal characteristic length 1.54 mm

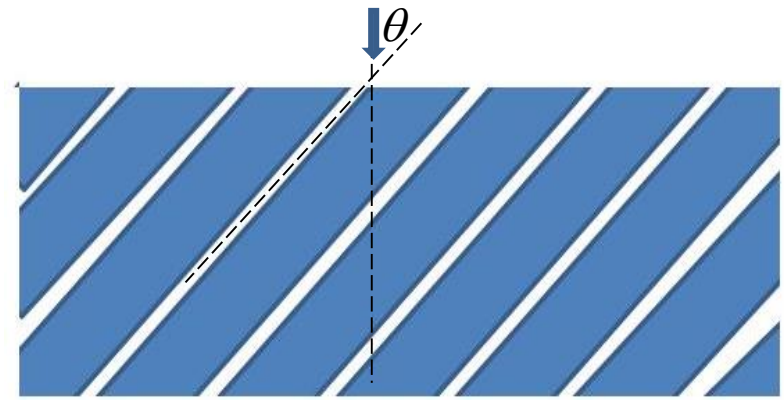


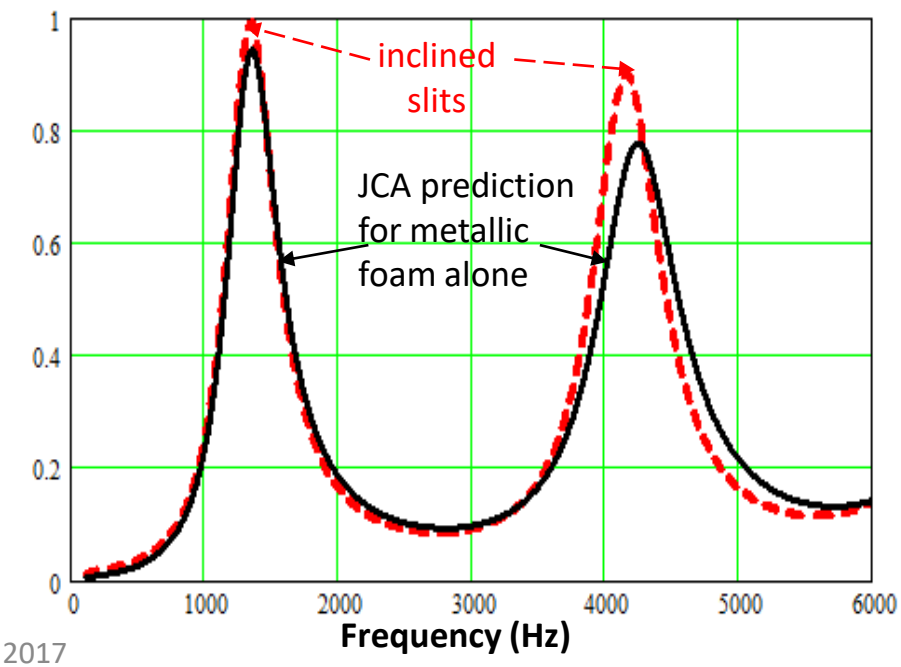




0.025 m thick simple microstructure - inclined slits

porosity 0.283, slit width 0.46 mm,
edge-to-edge spacing 1.27 mm, angle $\theta = 65°$
$\Rightarrow$ flow resistivity 17.28 kPa s m^{-2}, tortuosity 4.69

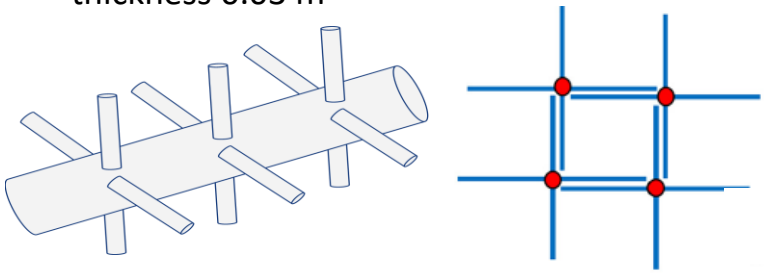


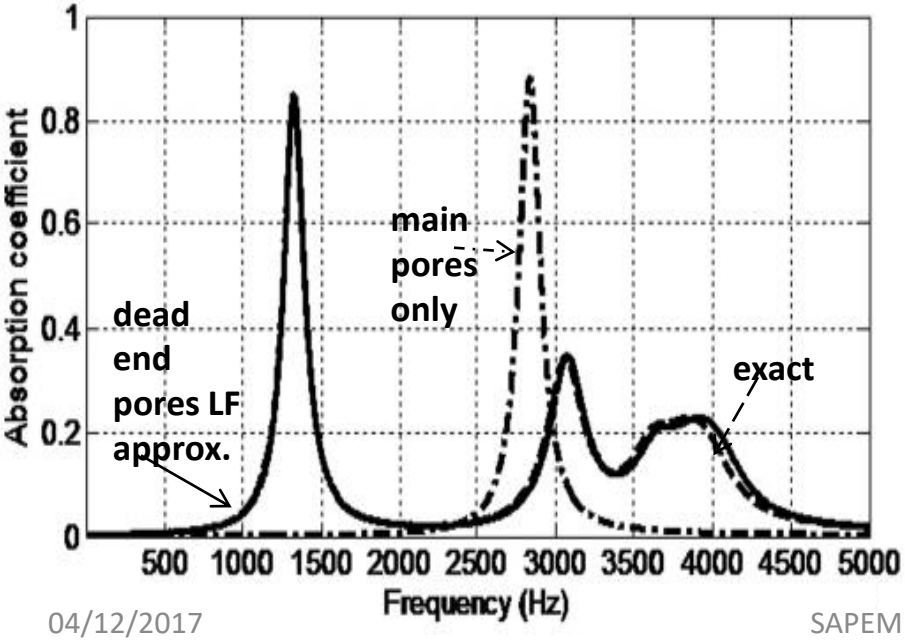


0.03 m thick layer with dead end pore microstructure

from Leclaire *et al* JASA 2015

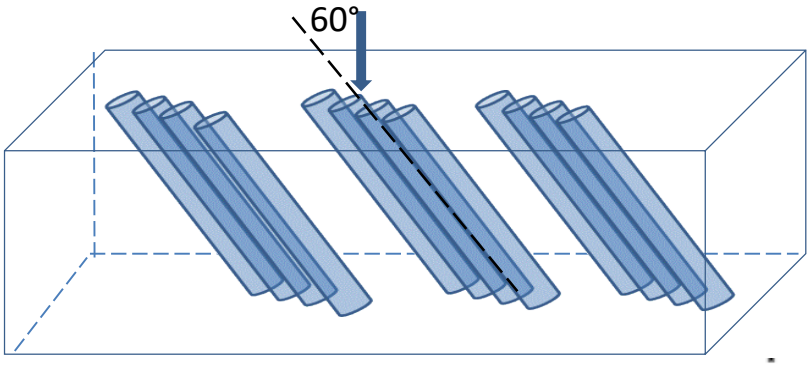
surface porosity 0.0169
main pore radius 1.5 mm
dead end pore radius 1 mm
dead end pore length 20 mm
N = 4 dead ends per period
thickness 0.03 m

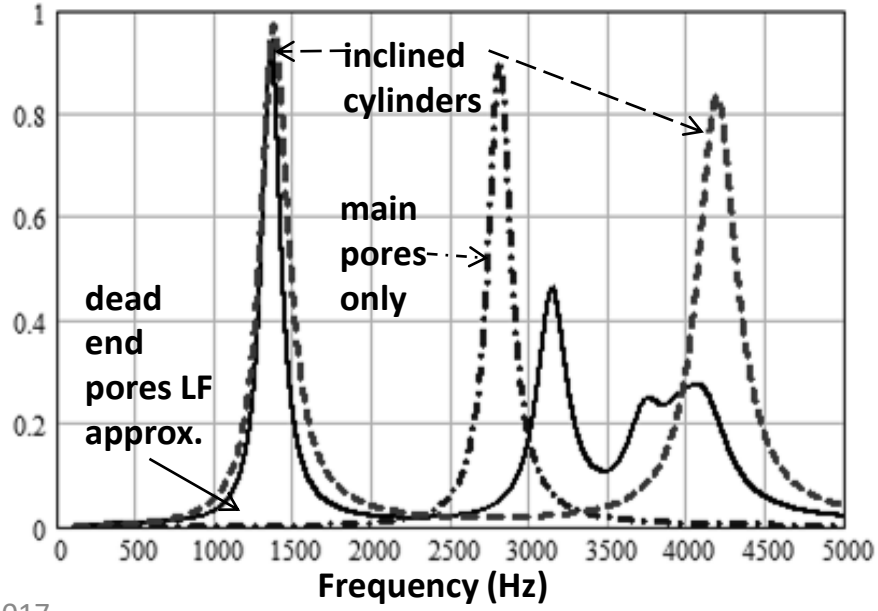




simple microstructure - inclined cylindrical pores

Bulk porosity 0.1
Identical pores radius 1 mm
thickness 0.03 m





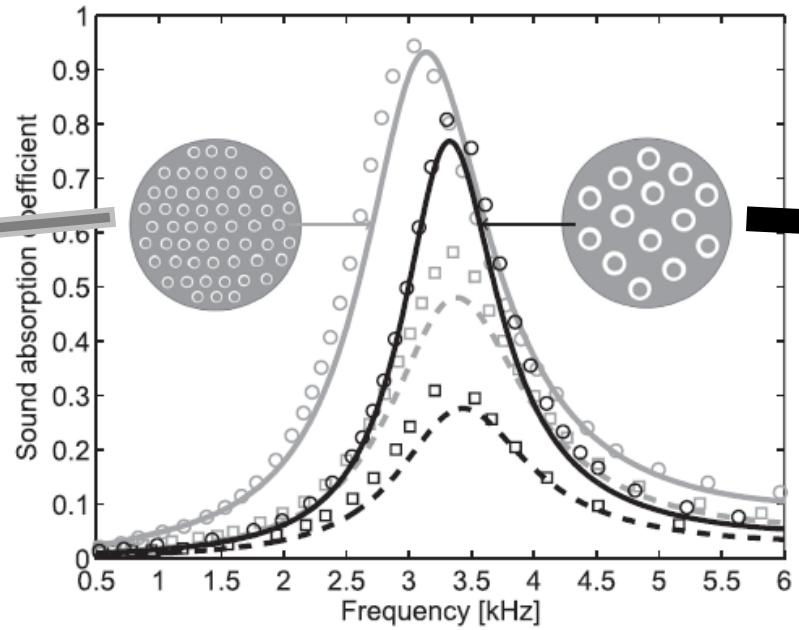
Annular cylindrical pores

Nori and Venegas
JASA 141 (2017)

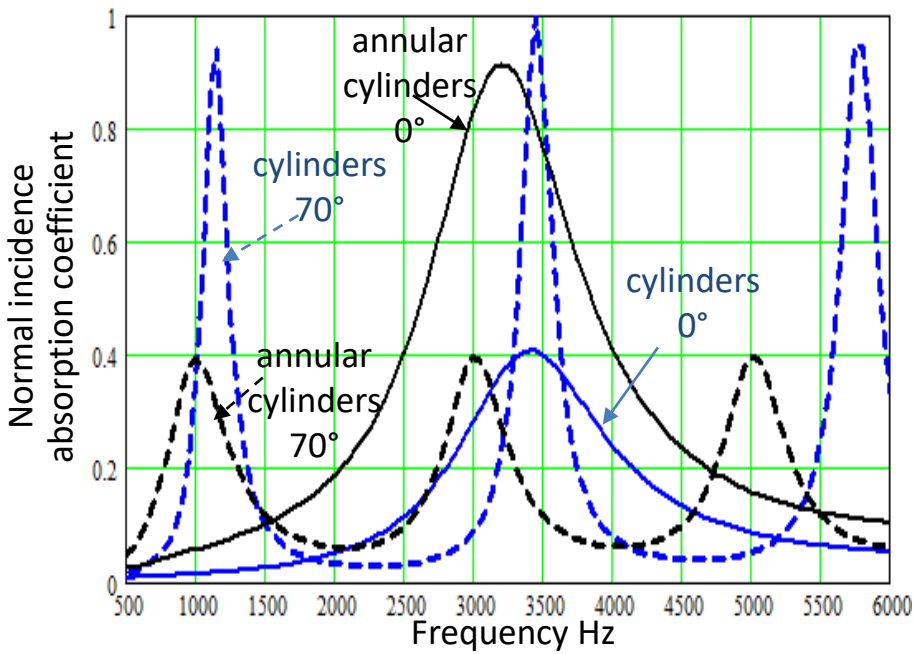
The Open University

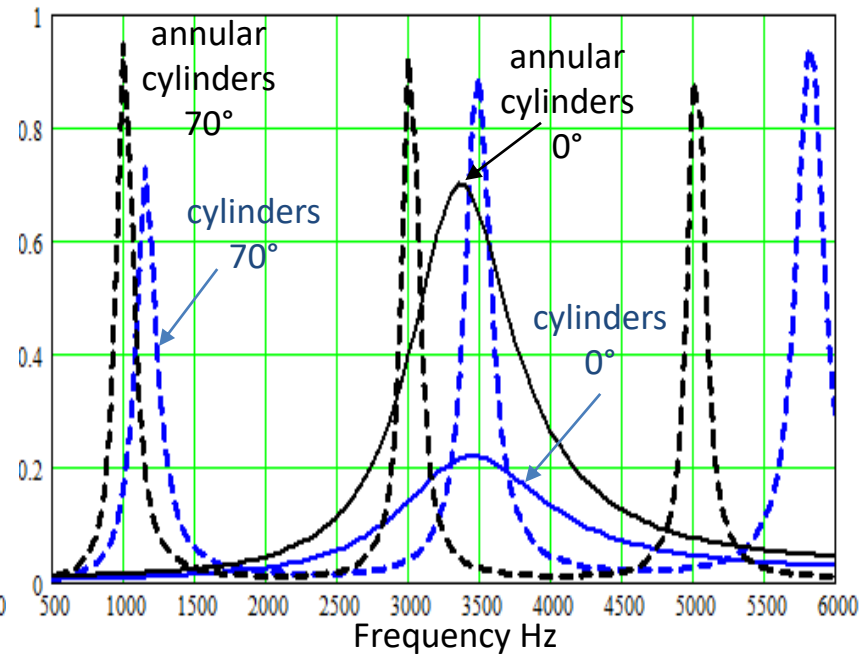
layer thickness 25 mm

annular pores -
outer radius 1 mm
inner radius 0.75 mm
porosity 0.1144;
cylindrical pores
radius 1 mm,
porosity 0.2616



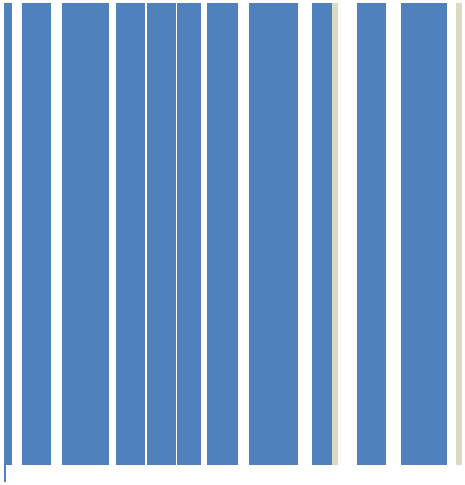
annular pores -
outer radius 2 mm
inner radius 1.25 mm
porosity 0.1623;
cylindrical pores
radius 2 mm,
porosity 0.2663



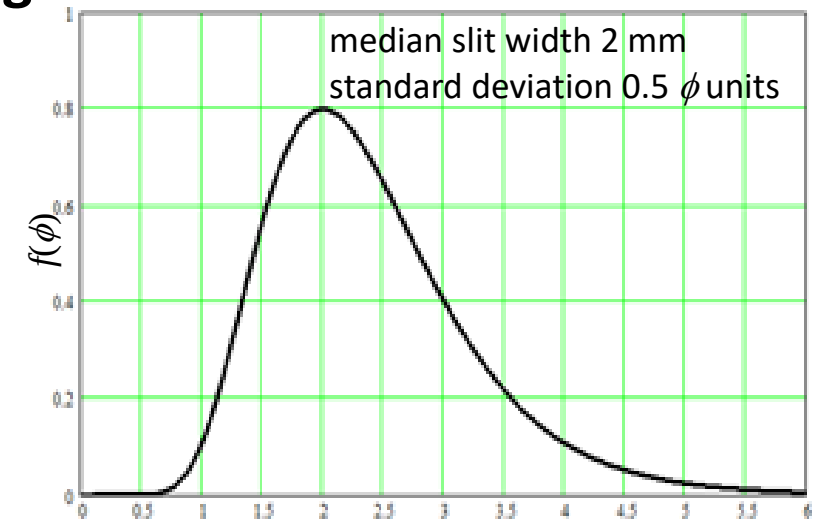


log-normal distributions of slit widths

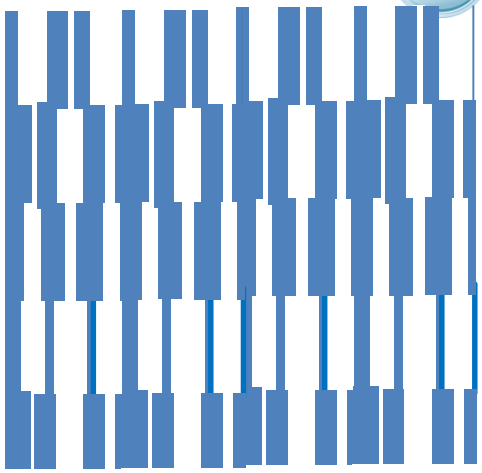
uniform



Yamamoto and Turgut, JASA 1998

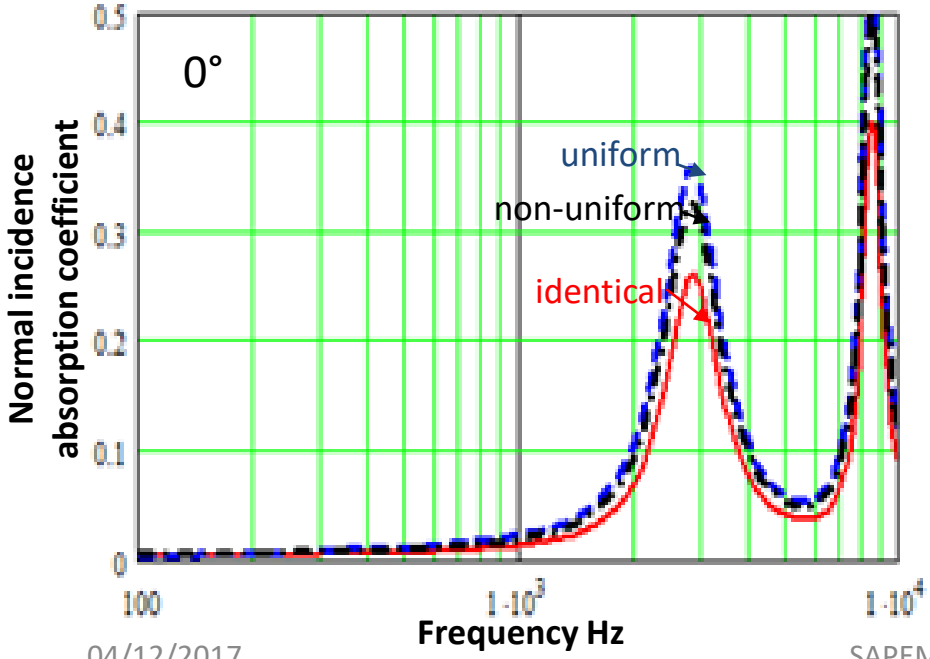


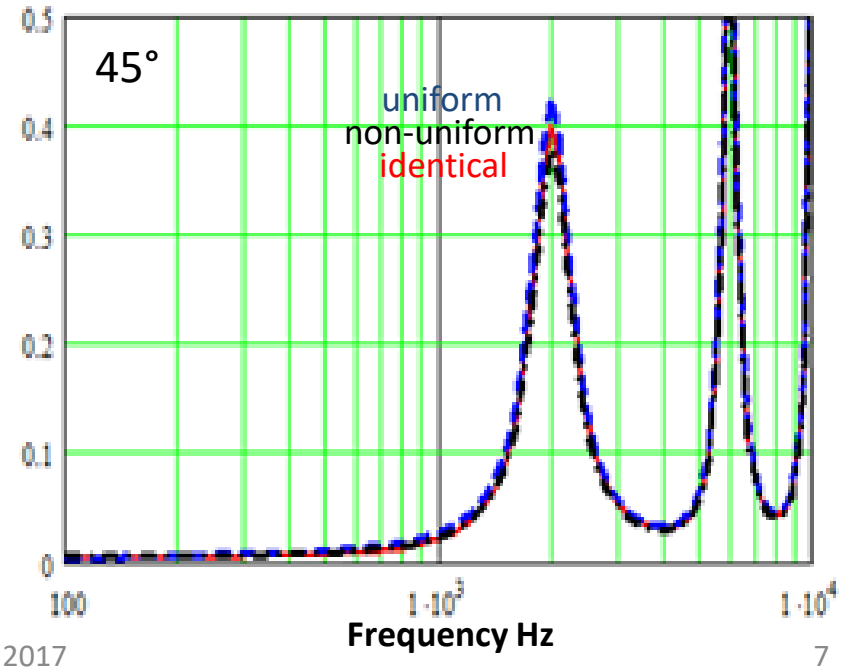
non-uniform



Horoshenkov *et al*, JASA 2016

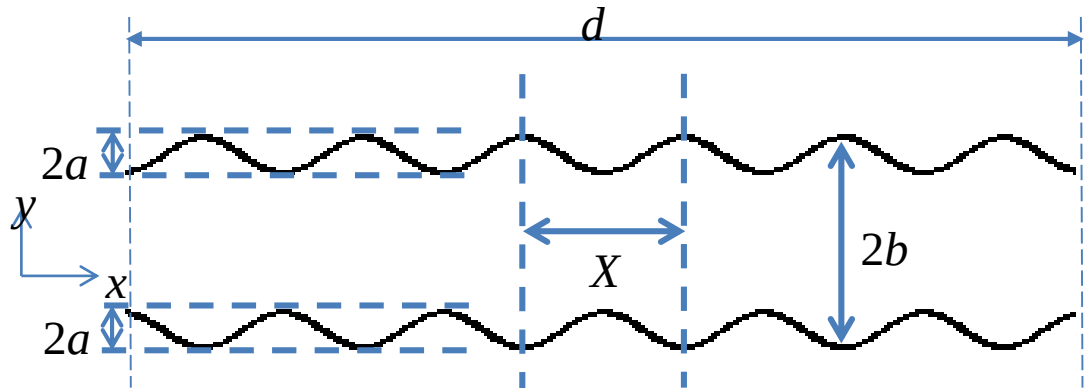
3-cm thick hard-backed rigid porous layer with porosity 0.3





Slits with sinusoidal sides

Song *et al* J. Appl. Phys. D. 2017; 50: 235303



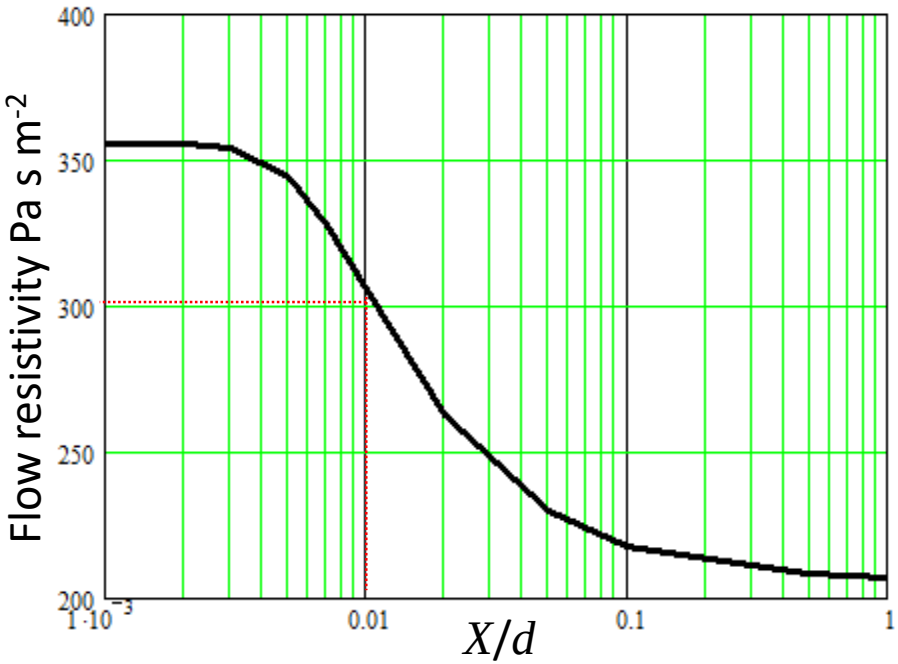
constraints:

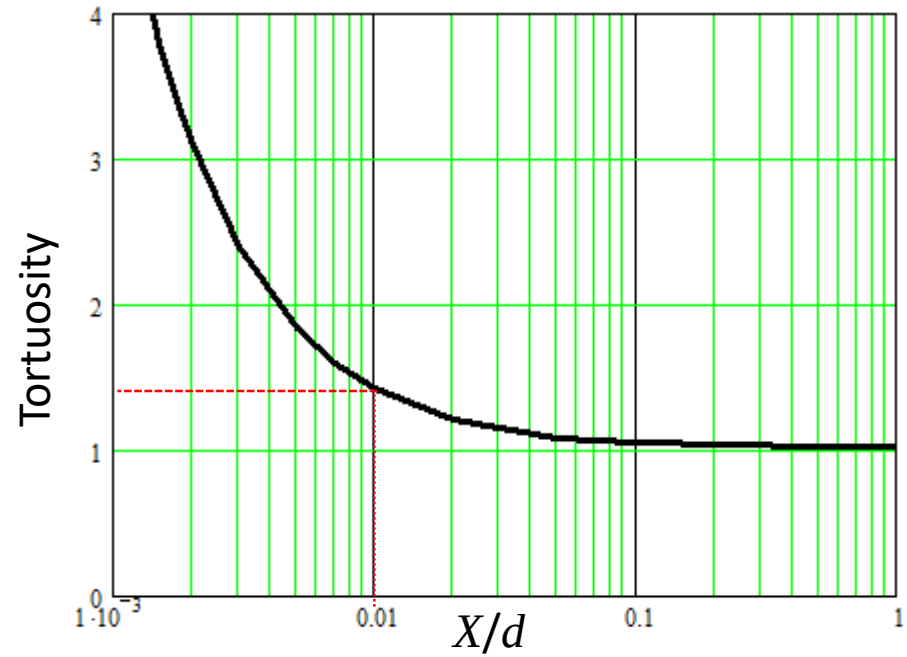
$a/2b \leq 0.1$

$X > a$

$a > 0.1$ mm

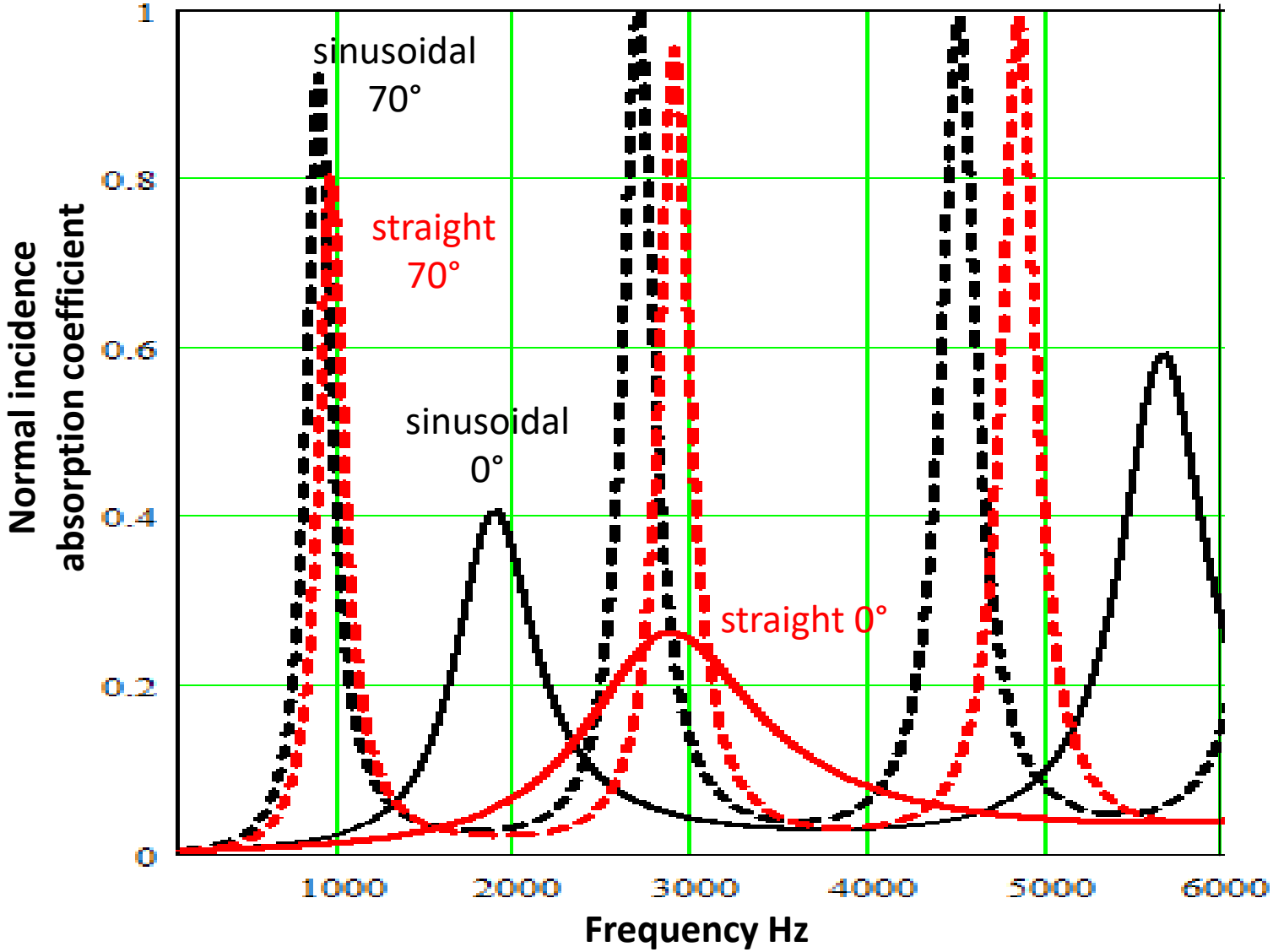
a = 0.2 mm, b = 1 mm, Ω = 0.3





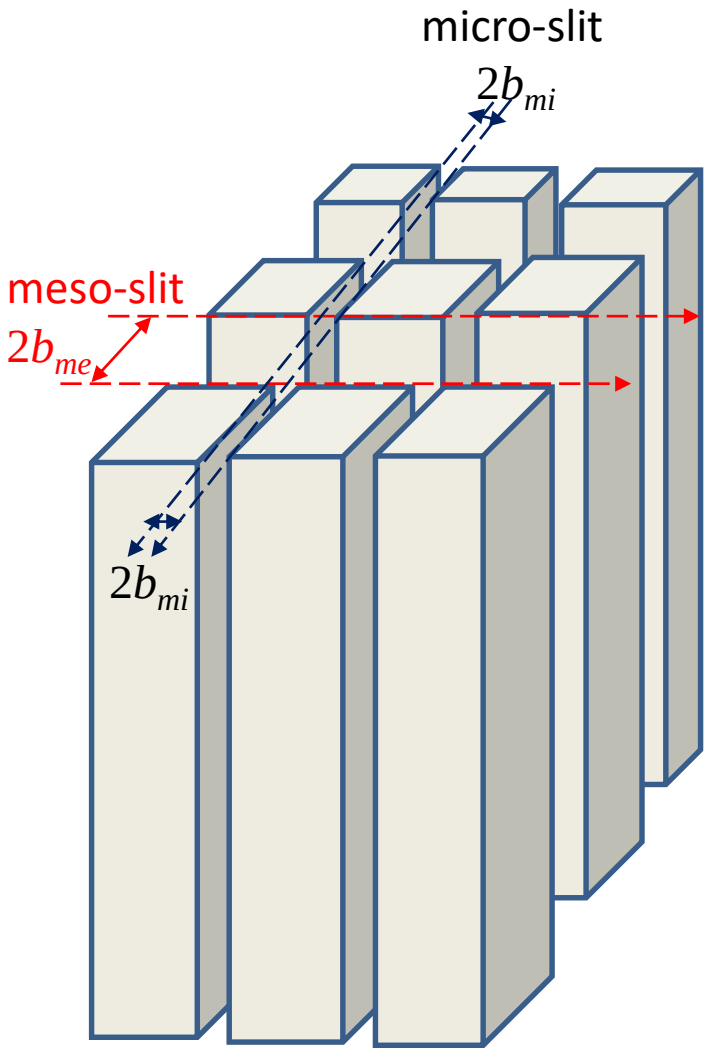
sinusoidally-sided *and inclined* slits

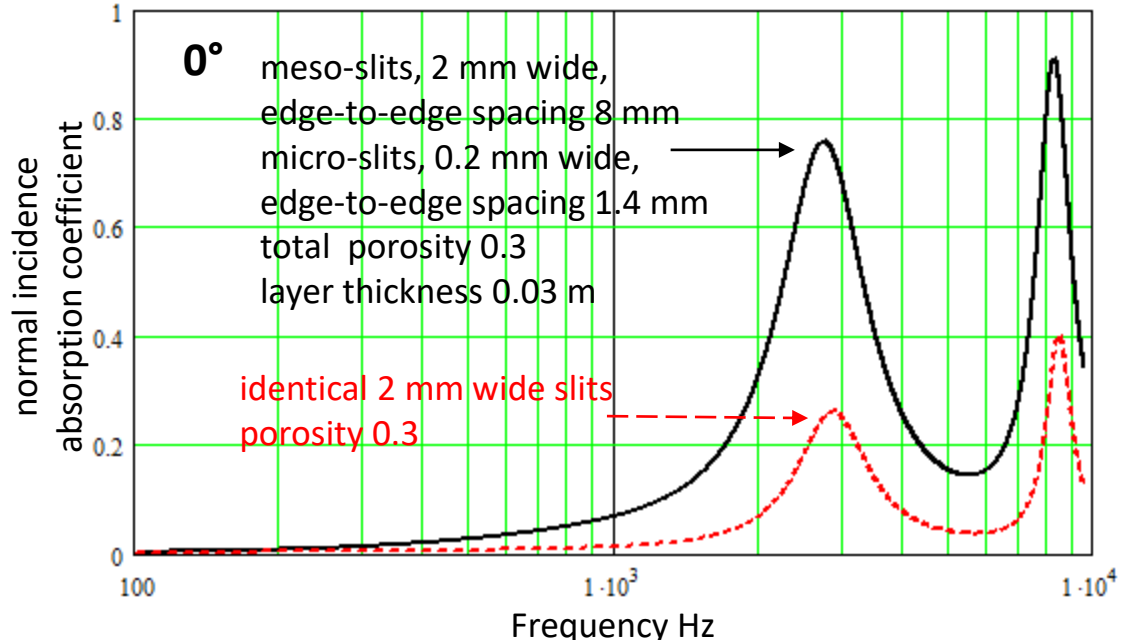
X = 0.3 mm, d = 30 mm, a = 0.2 mm, b = 1 mm, porosity 0.3

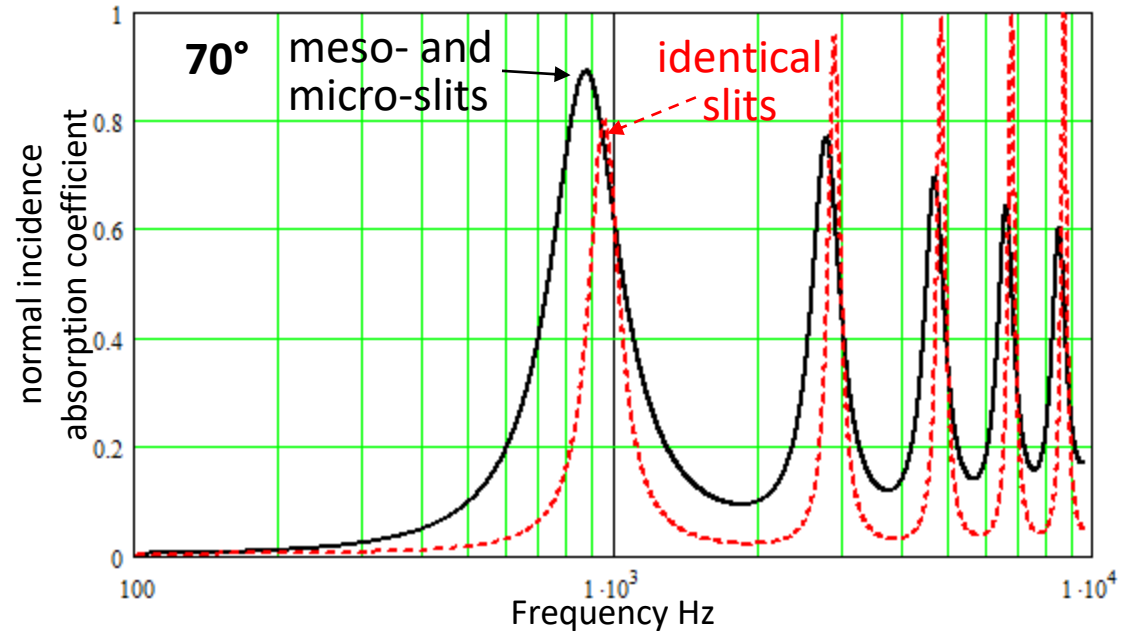


dual-porosity slits

Ren *et al* Chinese Physics Letters. 2015; 32: 014302







Summary of predicted quarter wavelength resonances of 0.03 m thick layers with porosity 0.3 for various microstructures

Microstructure	Frequency kHz	amplitude
Identical slits width 2 mm @ 0°	2.95	0.27
Identical slits width 2 mm @ 70°	0.95	0.8
Identical cylinders diameter 2mm @ 0°	2.95	0.47
Identical cylinders diameter 2mm @ 70°	0.95	0.95
Sinusoidal slits width 2 $\pm$ 0.4 mm @ 0°	2.8	0.28
Sinusoidal slits width 2 $\pm$ 0.4 mm @ 70°	0.8	0.95
Dual porosity slits widths 2 and 0.2 mm @ 0°	2.7	0.75
Dual porosity, sinusoidal meso-slits @ 0°	2.3	0.85
Dual porosity - slit widths 2 and 0.2 mm @ 70°	**0.85**	**0.9**

CONCLUSIONS

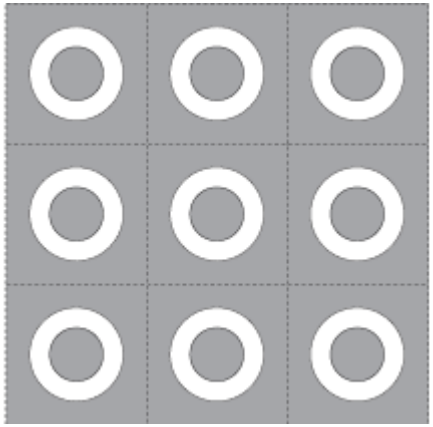
- The frequency of the quarter wavelength resonance of a hard-backed layer with low values of porosity and flow resistivity is decreased by increasing tortuosity.
- Tortuosity is increased by inclining pores with respect to the surface normal, by varying pore widths in a sinusoidal manner, by a dual porosity arrangements and by combinations of these.
- Identical inclined pores give comparable absorption spectra to those due to comparatively complicated microstructures.
- If the pores are annular and cylindrical or have log-normal width distributions, inclining the pores lowers the quarter wavelength resonance by less than other methods.
- Useful low frequency resonant absorption can be achieved by materials with simple microstructures suitable for manufacture by 3D printing.

cylindrical annular pores

Nori and Venegas JASA 141 (2017)

Johnson-Champoux-Allard-Lafarge formulation

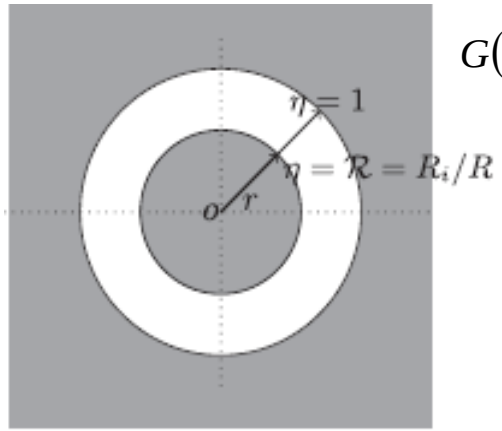
viscous and thermal characteristic lengths and permeabilities are equal



$$\rho_b(\omega) = T\rho_0\left[1 + \frac{iR_A\Omega}{\omega\rho_0 T} G(R, r, L, T)\right]$$

outer radius — radius ratio — centre-to-centre spacing

$$C_b(\omega) = (\gamma P_0)^{-1}\left[\gamma - (\gamma - 1)\left[1 + \frac{iR_A\Omega}{\omega\rho_0 T N_{PR}} G'(R, r, L, T)\right]^{-1}\right]$$



$$G(R, r, L, T) = \sqrt{\left(1 - \frac{4iT\eta\rho_0\omega}{R_A{}^2 R^2 (1-r)^2 \Omega^2}\right)}$$

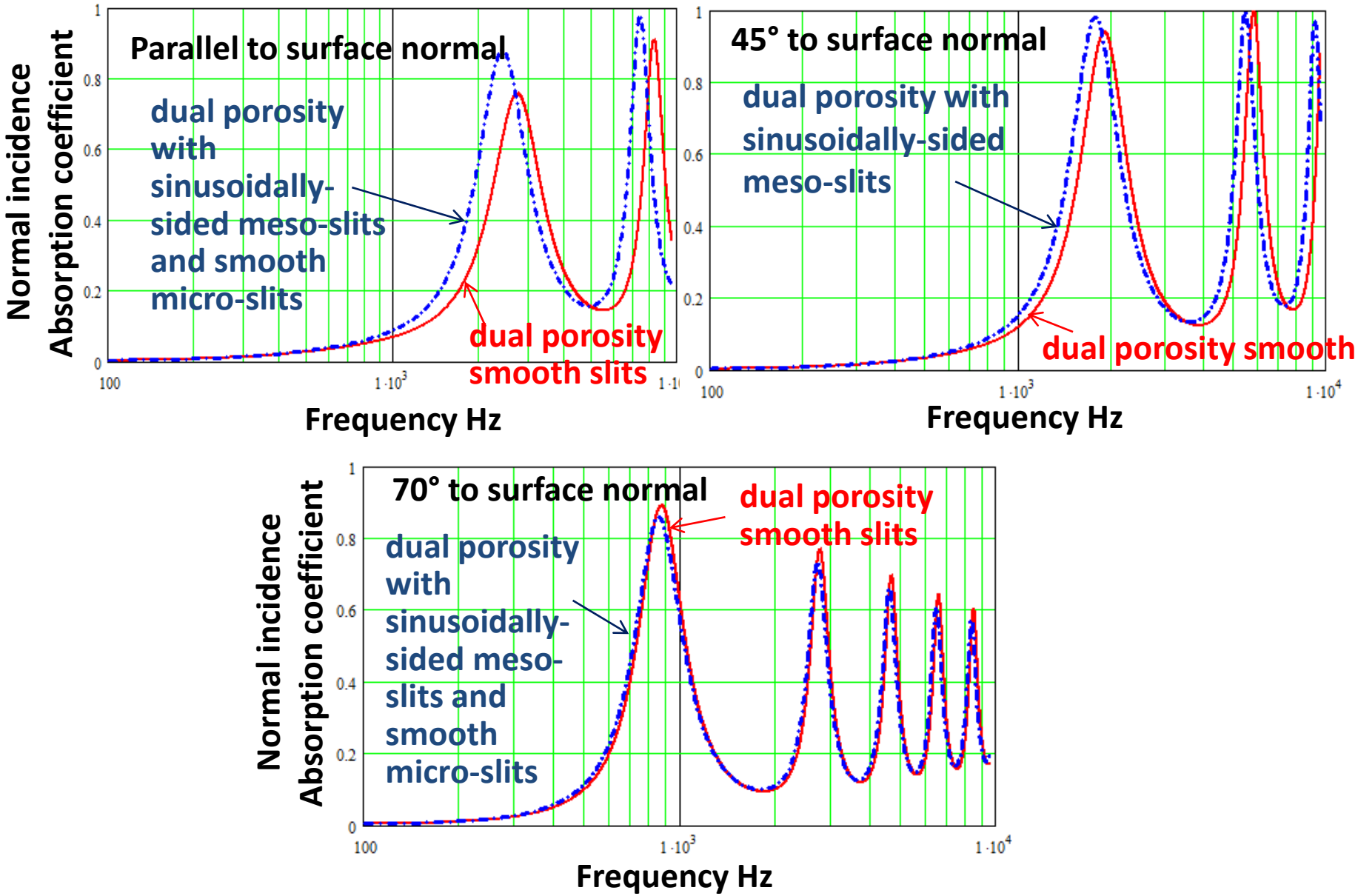
$$G'(R, r, L, T) = \sqrt{\left(1 - \frac{4iT\eta\rho_0\omega N_{PR}}{R_A{}^2 R^2 (1-r)^2 \Omega^2}\right)}$$

$$\Omega = \pi R^2 (1 - r^2)/L^2$$

$$Y(r) = 1 + r^2 - (1 - r^2)/ln(1/r)$$

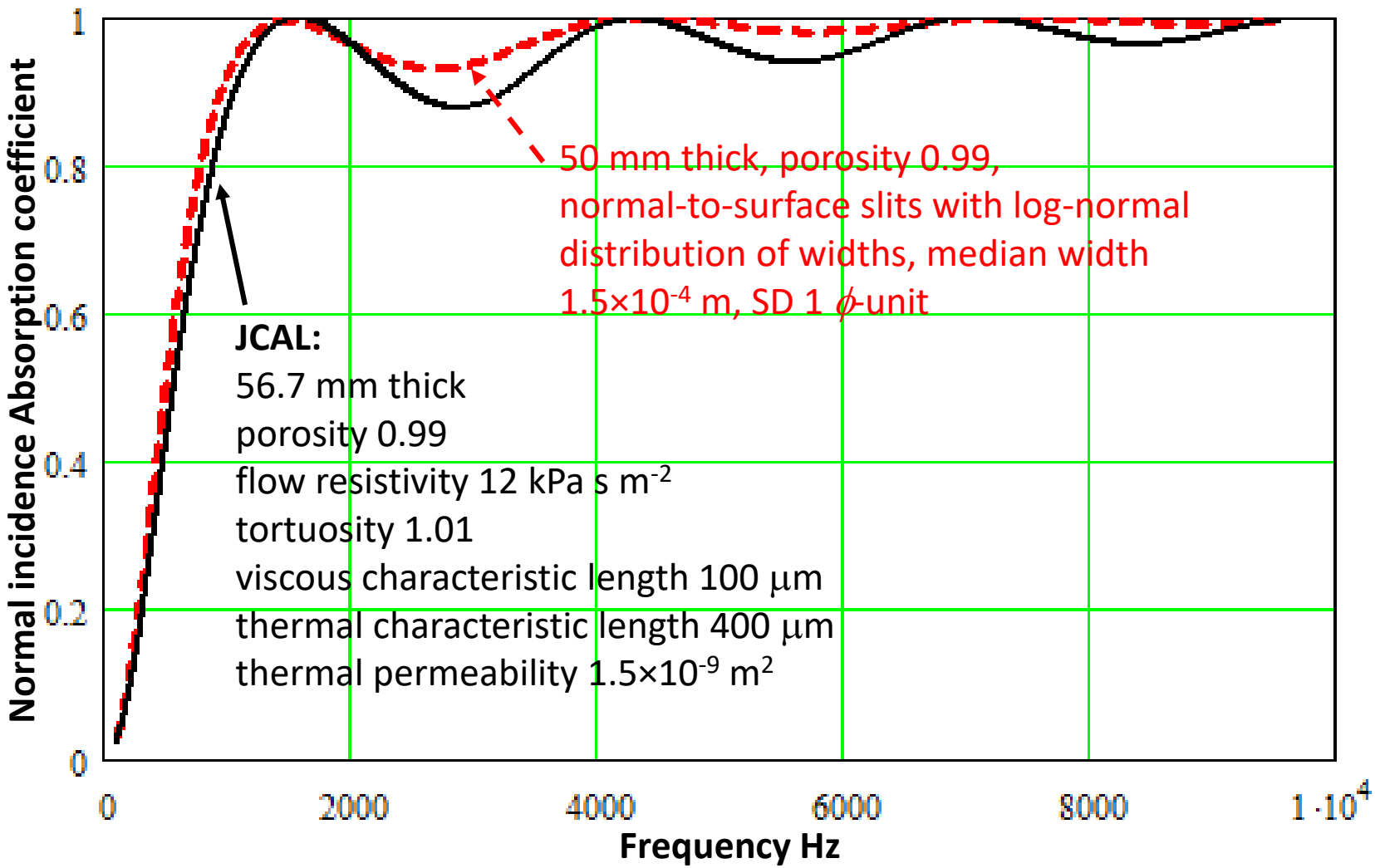
$$R_A = \frac{8T\eta}{\Omega R^2 Y(r)}$$

Dual porosity: meso-pores with sinusoidally-varying sides

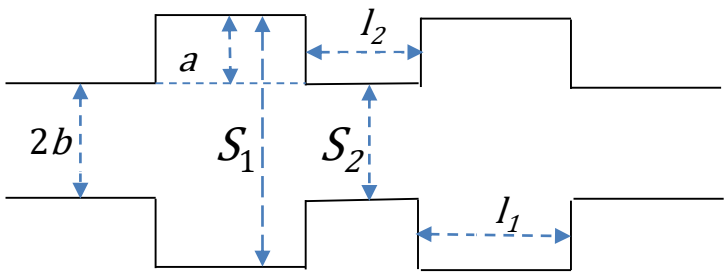


Melamine foam

Chevillotte *et al* JASA 128 1775 (2010)



Slits with regular discrete width variation



Equation (5.A.7) p.104 in Allard and Atalla text

$$T = \frac{(l_1 S_1 + l_2 S_2)(l_2 S_1 + l_1 S_2)}{(l_1 + l_2)^2 S_1 S_2}$$

Assumes constant velocities in each section

$$S_1 = 2(a + b),\ S_2 = 2b$$

a/b (b = 0.001 m)	l_2/l_1	tortuosity
0.2	0.1	1.003
2	0.9	1.66
4	0.9	2.59
8	0.9	4.54